

## POSITIO IMPOSSIBILIS AND CONCEPT FORMATION: DUNS SCOTUS ON THE CONCEPT OF INFINITE BEING\*

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### 0. Introduction

0.1 Scotus made important attempts to formulate an acceptable version of “infinite being” (*ens infinitum*), a concept that plays a distinguished role in his metaphysics and natural theology<sup>1</sup>. The *intelligibility* of the concept, particularly in the analysis of the full compatibility of ‘being’ and ‘infinite’, is something that he pursued especially in the proof of the existence of God<sup>2</sup>. Scotus usually considered these efforts no better than sophisticated persuasions which served to clarify that there is no incompatibility between entity and infinity, that there is no simple convertibility between entity and finity, and finally that the entity compatible with infinity in quantity is *a fortiori* also compatible with infinity “in the perfection that exists at the same time”<sup>3</sup> – i.e., with a certain kind of actual infinity in entity<sup>4</sup>. In his *Quodlibet* 5 (1306/1307), the Subtle Doctor seems to achieve a better result. There, in the analysis and construction of the concept of *ens infinitum*, he

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<sup>1</sup> “Infinite being” appears in *Ordinatio* prol. p. 3 q. 1-3 n. 168, ed. Vat., vol. I, Civitas Vaticana 1950, p. 110-112, as the most perfect concept that the human intellect can have of God in the present state, i.e., of the first object of “theology as such” (*theologia in se*). For this reason, it functions for Scotus as the first subject of “our theology” (*theologia nostra*).

<sup>2</sup> See especially Duns Scotus, *Ordinatio* I d. 2 p. 1 q. 1-2 n. 1-156, ed. Vat., vol. II, Civitas Vaticana, 1950.

<sup>3</sup> For discussion of Scotus’s proof *via eminentiae* that the First Being in the essential order of causes is infinite, see *Ordinatio* I d. 2 p. 1 q. 1 n. 131-139 p. 206-210; *De primo principio* IV concl. 9 n. 78-79, ed. W. Kluxen, Wissenschaftliche Buchgesellschaft, Darmstadt, 1974, p. 102-106.

<sup>4</sup> See the recent and fine exposition of L. Honnefelder, *Duns Scotus*, München, Beck Verlag, 2005, p. 91-102. Expositions of Scotus’s main arguments for the infinity of the First Cause in the context of his proof of the existence of God are also offered by R. Cross, *Duns Scotus on God*, Aldershot, Ashgate, 2005, p. 91-114, and G. Sondag, *Duns Scot. La métaphysique de la singularité*, Vrin, Paris, 2005, p. 107-120.

makes use of the technique of *positio impossibilis*. In this paper I want to present and assess this later effort.

**0.2** I shall not evaluate here *all* of Scotus's achievements concerning the possibility of that concept, nor explore, for example, how "infinity" operates as a mode<sup>5</sup> in order to coordinate all predications of God as a subject of propositions<sup>6</sup>. My focus is on the strategy for the formation of the concept of *ens infinitum*, starting with a very briefly stated account in the *Physica* of the potential infinite according to Aristotle (I), and proceeding from there to develop the hypothesis that Scotus utilizes *positio impossibilis* in the "imagination" of an actual quantitative infinity (II) and in the "imagination" of an actual infinity in entity (III). I describe the technique in *Quodlibet* 5 up to the point where it is clear or at least apparent that Scotus continues to employ it. I think it is possible to draw an account of the technique from what I call the "descriptive phase" of the argument, where infinity in entity – as a consequence of a *positum* – is treated more and more as a mode of being (IV- VI). I conclude with remarks on what *positio impossibilis* might be theoretically for Scotus, using this reading as a case study for his understanding of *ars obligatoria* (VII).

## I. Aristotelian Potential Infinite

**I.1** At the start of *Quodl.* 5 n. [2] 5, Scotus mentions Aristotle's definition of the infinite (of the *apeiron* or the "unlimited") as formulated in *Physica* III 6, 207a7-9 (206b32-207a15)<sup>7</sup>. Here the infinite, understood under the category of quantity, is such that, no matter how much has been already taken, always a part outside it remains to be taken<sup>8</sup>. The infinite

<sup>5</sup> Starting from the construction of the concept of ontological infinity in *Quodlibet* 5, I attempt a theoretical analysis of the intrinsic mode in: R. H. Pich, *Infinity and Intrinsic Mode*, in: R. H. Pich (ed.), *New Essays on Metaphysics as Scientia Transcendens*, Louvain-la-Neuve, FIDEM, 2007, p. 159-214.

<sup>6</sup> This is the thesis successfully defended by F. J. S. Catania, *John Duns Scotus on Ens Infinitum*, in: *The American Catholic Philosophical Quarterly*, 68 (1993), p. 37-54. See also J.-M. Counet, *L'univocité de l'étant et la problématique de l'infini chez Jean Duns Scot*, in: J. Follon et J. McEvoy (eds.), *Actualité de la pensée médiévale*, Éditions de l'Institut Supérieur de Philosophie – Éditions Peeters, Louvain-la-Neuve – Louvain-Paris, 1994, p. 314f.

<sup>7</sup> For Aristotle's theory of the infinite, see A. F. Koch, *apeiron / unendlich*, in: O. Höffe (Hrsg.), *Aristoteles-Lexikon*, Redaktion: Rolf Geiger und Philipp Brüllmann, Alfred Kröner Verlag, Stuttgart, 2005, p. 55-58, is to be found essentially in *Physica* III 4-8. On the potential infinite, see especially *Metaphysica* IX 6, 1048b9-17, and *Physica* III 6, 206a18ff.

<sup>8</sup> See also Aristotle, *Physica*, translated by R. P. Hardie and R. K. Gaye, in: Aristotle, *The Works of Aristotle*, translated into English under the editorship of W. D. Ross, Vol. II, Clarendon Press, Oxford, 1970 (first edition 1930), III 6, 207a7-14: "A quantity is infinite if it is such that we can always take a part outside what has been already taken. On the other hand, what has nothing outside it is complete and whole. For thus we define

is not that out of which there is nothing, but rather that out of which there is always something (*Physica* III 6, 207a1-2). It should be noted that the point at issue for Aristotle is not to know about the existence of the infinite in mathematical or merely abstract things, but to judge, in a general sense, whether there can be *actual* infinite continuous magnitudes in nature; as is well known, Aristotle's answer is negative – see *Physica* III 5-6<sup>9</sup>. Following that conception, Scotus affirms that the infinite in quantity can only be “in potency” (*in potentia*), so that in it some new thing of a corresponding dimension can always be taken in *only* after something else<sup>10</sup>. On this view, whenever this is done, a new extensive “magnitude” (*magnitudo*) is caused<sup>11</sup>. And it is a consequence of the ceaseless acceptance of some further part (through addition, division, or both) that the quantitative infinite *is*, in actuality, only finite – that the “magnitude infinite in extension” does not exist *in actuality*. In other words, such an infinite is, *in actu*, “a certain part of the potential infinite whole” (*quaedam pars totius infiniti potentialis*), and therefore remains something outside this same infinite that can still be taken into it<sup>12</sup>.

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the whole – that from which nothing is wanting (...) (...) – the whole is that of which nothing is outside. On the other hand that from which something is absent and outside, (...) is not ‘all’. ‘Whole’ and ‘complete’ are either quite identical or closely akin. Nothing is complete (*teleion*) which has no end (*télos*); and the end is a limit”. As far as the Aristotelian potential infinite is concerned (*Physica* III 6, 206a18), this definition of the infinite is compatible with both forms of related potentiality, namely through “addition” (*prosthesis*) and “division” (*dihairesis*); see *Physica* III 6, 206a15f. See also A. F. Koch, *apeiron / unendlich*, in: O. Höffe (Hrsg.), op. cit., p. 57f. For some of the central passages of Aristotle's *Physica* concerning infinity, with short comments, see Th. Heath, *Mathematics in Aristotle*, Thoemmes Press, Bristol, 1998 (reprinted from the 1949 edition), p. 102-113.

<sup>9</sup> See G. Sondag, *Jean Duns Scot sur l'infini extensif et l'infini intensif*, in: *Revue Thomiste* 105 (2005), p. 119; A. F. Koch, *apeiron / unendlich*, in: O. Höffe (Hrsg.), op. cit., p. 55-56. See also W. D. Ross, *Commentary*, in: Aristotle, *Aristotle's Physics*, a revised text with introduction and commentary by W. D. Ross, Clarendon Press, Oxford, 1936 (reprinted 1979), p. 364-366; R. Mondolfo, *O infinito no pensamento da Antiguidade Clássica*, Mestre Jou, São Paulo, 1968, p. 429-461; Th. Kouremenos, *Aristotle on Mathematical Infinity*, Franz Steiner Verlag, Stuttgart, 1995, p. 9-34, 53-62. On the spatial-temporal infinity of the cosmos in today's physics, see the remarks in J. Leslie, *Finite/infinite*, in: J. Kim and E. Sosa (eds.), *A Companion to Metaphysics*, Blackwell Publishers, Oxford, 1995 (reprinted 1998), p. 174. See also A. W. Moore, *Infinity*, in: E. Craig (ed.), *The Routledge Encyclopedia of Philosophy*, Routledge, London – New York, 1998, Vol. 4, p. 772-778.

<sup>10</sup> See Duns Scotus, *Quaestiones quodlibetales* q. 5 n. [2] 5, Ed. by F. Alluntis, Madrid, 1968, p. 167: “De primo: ‘Infinitum’, secundum Philosophum III *Physicorum*, ‘est cuius quantitatem accipientibus, id est, quantumcumque accipientibus, semper aliquid restat accipere’; et ratio est: quia infinitum in quantitate, sicut loquitur Philosophus, non potest habere esse nisi in potentia, accipiendo semper alterum post alterum; (...)”.

<sup>11</sup> See F. Lychetus, *Commentarius*, in: Duns Scotus, *Quaestiones quodlibetales* q. 5 n. 1, Ed. by L. Wadding, vol. XII, Lyon, 1639 (repr. Georg Olms, Hildesheim, 1969), p. 119: “(...) scilicet accipiendo ab vna magnitudine, & addendo alteri magnitudini, quia in actu non inuenitur magnitudo infinita extensue. Siue etiam semper nouam, & nouam magnitudinem causando & addendo semper”.

<sup>12</sup> See Duns Scotus, *Quaestiones quodlibetales* q. 5 n. [2] 5 p. 167: “(...) et ideo quantumcumque accipiatur illud non est nisi finitum et quaedam pars totius infiniti potentialis, et ideo restat aliquid alterum ipsius infiniti accipiendum”.

**I.2** It is important to recognize that, already in the first step of constructing this concept, Scotus directs the very short exposition of the Aristotelian account towards *two* notions that pertain analytically<sup>13</sup> to the concept he intends to form: after all, such a 'finite' in actuality and 'infinite' in potency cannot be either (i) a whole thing or (ii) a perfect thing. The consequences of the Aristotelian definition, he explains, are that, on the one hand, the quantitative infinite according to potentiality (i') has only an *esse in fieri*, a kind of reality that is realized "in a becoming or in a process"<sup>14</sup>; the quantitative infinite is not able to fulfill "the concept of a whole" (*rationem totius*), since it belongs to the definition of "whole" (*totum*) that there not be anything outside it. On the other hand, (ii') the quantitative infinite is also incapable of satisfying the concept of something «perfect» (*nec est perfectum*), since the concept of perfect is such that "the perfect is that to which nothing of [the corresponding] perfection is lacking"<sup>15</sup>.

## II. Imagination of an Actual Quantitative Infinity

**II.3** In the second step of the argument, Scotus develops the conceptual structure of 'intensive infinity', introducing the "imagination" (*imaginatio*) of an *actual* quantitative infinity. In the process of 'imagination', we replace completely (*commutemus*) all conceivable parts of a given quantity infinite in potentiality (i.e., one that increases in magnitude only through the acceptance of a further part after the previous one) with "the concept of the infinite in actuality, [still] in quantity" (*in rationem infiniti in actu, in quantitate*), on the condition that the infinite quantity *could be* in actuality (*si posset ibi esse in actu*). The text makes clear that by 'imagination' Scotus is recommending the total replacement of one *ratio* of infinite quantity with another *ratio* of infinite quantity. To take all possible infinite parts of a quantity at once in actuality should mean conceptually that in the new notion there is as much infinite quantity in actuality as

<sup>13</sup> And, at least for Scotus, are not synonymous; see below under III.12 and III.14.

<sup>14</sup> See S. Wolf, *Das potentiell Unendliche – Die aristotelische Konzeption und ihre modernen Derivate*, Verlag Peter Lang, Frankfurt a. M., 1983, p. 20; W. Charlton, Aristotle's Potential Infinites, in: L. Judson (ed.), *Aristotle's Physics – A Collection of Essays*, Oxford, Clarendon Press, 1991, p. 129-149; L. Sweeney, *Divine Infinity in Greek and Medieval Thought*, Peter Lang, New York, 1992, p. 150f. 164-165. See Aristotle, *Physica*, translated by R. P. Hardie and R. K. Gaye, op. cit., III 6, 206b16-18: "By addition then, also, there is potentially an infinite, namely, what we have described as being in a sense the same as the infinite in respect of division. For it will always be possible to take something *ab extra*".

<sup>15</sup> See L. Honnfelder, *Scientia transcendens*, Felix Meiner Verlag, Hamburg, 1990, p. 110f. See Duns Scotus, *Quaestiones quodlibetales* q. 5 n. [2] 5 p. 167: "Ex hoc concludit quod, sicut infinitum habet esse in fieri et in potentia in quantitate, ita non habet rationem totius; quia 'totum' est cuius nihil est extra; sed extra illud infinitum, hoc est, extra illud quod habet esse de ipso, semper est aliquid extra; nec est perfectum, quia perfectum est cui nihil perfectionis deest; si isti semper aliquid deest". See Aristotle, *Physica*, translated by R. P. Hardie and R. K. Gaye, op. cit., III 6, 207a7-14 (see note 8).

there is in potentiality<sup>16</sup>. The second notion seems to correct the first in order to satisfy the concepts of 'whole' and 'perfect'.

II.4 At the same time, as was originally noted by G. Sondag, precisely at this point it should be stressed that the procedure of 'imagining'<sup>17</sup> – which departs from the *potential* infinite in extension but assumes, *if it were possible* (*si posset*), the infinite in extension as existing in *actuality* – resembles the procedure of *positio impossibilis*. It is today widely recognized that *positio impossibilis* is a procedure employed by Boethius in his *De consolazione philosophiae* and *De hebdomadibus*. The technique figures prominently in treatises *De obligationibus* of the 13<sup>th</sup> and 14<sup>th</sup> centuries, and William of Sherwood's *Obligationes* is most likely the historical source of Scotus's acquaintance with the logical tool<sup>18</sup>. Emphasizing the role of the logical analysis of concepts in the procedure<sup>19</sup>, I maintain, together with G. Sondag, that two conditions must be met in order for any use of the 'impossible hypothesis' or the *impossibile positum* to be acceptable<sup>20</sup>: (i) it shall not be contradictory in itself and (ii) the *respondens* in the disputation is not

<sup>16</sup> See Duns Scotus, *Quaestiones quodlibetales* q. 5 n. [2] 6 p. 168: "Ex hoc ad propositum; commutemus rationem infiniti in potentia in quantitate in rationem infiniti in actu in quantitate, si posset ibi esse in actu. Si enim nunc necessario semper cresceret quantitas infiniti per acceptionem partis post partem, sic et imaginaremur omnes partes acceptibiles esse simul acceptas vel simul remanere, haberemus infinitam quantitatem in actu, quia tanta esset in actu, quanta esset in potentia".

<sup>17</sup> In *Quodl.* q. 5 n. [2-4] 5-11, Scotus makes use of verbal forms of 'imaginare' and of the noun 'imaginatio' as well. In *Quodl.* q. 5 n. [3] 8 he uses the expression 'secundum imaginationem'. The technique of reasoning *secundum imaginationem*, which will be connected here to that of *positio impossibilis* (see below), is expressly mentioned by J. E. Murdoch, *Infinity and Continuity*, in: N. Kretzmann; A. Kenny; J. Pinborg (eds.), *The Cambridge History of Later Medieval Philosophy. From the Rediscovery of Aristotle to the Disintegration of Scholasticism 1100-1600*, Cambridge University Press, Cambridge, 1982, p. 566-567, as an instrument of logical analysis of concepts such as omnipotence, absolute power, the infinite, and the continuous.

<sup>18</sup> See A. Vos, *The Philosophy of John Duns Scotus*, Edinburgh, Edinburgh University Press, 2006, p. 196-222.

<sup>19</sup> The purpose of *positio impossibilis* was first of all the logical training on consequences proceeding in accordance with accepted rules. On the rules of *positio* as the most common type of obligations disputation (i.e., exercise in 'technical' norms of logical consistency), see for instance S. Knuuttila, *Positio impossibilis* in Medieval Discussions of the Trinity, in: C. Marmo (ed.), *Vestigia, imagines, verba. Semiotics and Logic in Medieval Theological Texts (XIIIth-XIVth. Century)*, Brepols, Turnhout, 1997, p. 277-279.

<sup>20</sup> That is, in order to adhere correctly to the main rule of the *ars obligatoria*, which affirms that, hypothetically, "Anything can be assumed", remembering that *positio impossibilis* is a specific view of that technique; see A. Vos, op. cit., p. 210f. See also E. J. Ashworth, *Ralph Strode on Inconsistency in Obligational Disputations*, in: K. Jacobi (Hrsg.), *Argumentationstheorie. Scholastische Forschungen zu den logischen und semantischen Regeln korrekten Folgerns*, E. J. Brill, Leiden, 1993, p. 363-386 (here p. 364-366). Updated studies on medieval obligations can be found in M. Yrjönsuuri (ed.), *Medieval Formal Logic*, Kluwer, Dordrecht, 2001; for a survey of the literature and interpretations see also C. Dutilh Novaes, *Medieval Obligationes as Logical Games of Consistency Maintenance*, in: *Synthese*, 145:3 (2005), p. 373-376.

authorized to invoke the principle that anything whatever follows from the impossible (*ex impossibili quodlibet*)<sup>21</sup>. According to rules (i) and (ii), an *impossibile positum* expresses something logically conceivable that has no existence *naturaliter* (a hypothesis that is impossible *naturaliter*, for example, the idea that a quantitatively infinite continuity is a state of affairs that exists in actuality). And it is important to recall that rules of *positio impossibilis*, as S. Knuuttila has stressed, were frequently used to clarify theological matters – such as the Trinity – and that those who employed them in such matters were required to admit particular hypotheses that are conceivable logically even when they are inconceivable or impossible *doctrinally*, insofar as they are at odds with orthodox Christian theology<sup>22</sup>. Be that as it may, whatever our study concludes about the nature of *positio* techniques, it will employ the rules of *positio impossibilis* to clarify metaphysical matters and not theological ones. As it stands, there are studies on Scotus's application of the *ars of positio impossibilis* to questions about the persons of the Trinity<sup>23</sup>.

**II.5** It is my view, at least provisionally, that in this manner of transferring an analysis from the «realm of the physically possible (...) to the broader realm of the logically possible», a specific *positum* helps to define the nature of some thing – here of an infinite being. In order to help in this way, the *positum* must be conceivable without contradiction by means of a

<sup>21</sup> See G. Sondag, Jean Duns Scot sur l'infini extensif et l'infini intensif, op. cit., p. 120 note 33; also Ch. J. Martin, Impossible *positio* as the Foundation of Metaphysics or, Logic on the Scotist Plan?, in: C. Marmo (ed.), *Vestigia, imagines, verba. Semiotics and Logic in Medieval Theological Texts (XIIth.-XIVth. Century)*, Brepols, Turnhout, 1997, p. 258: "Scotus' account of this kind of impossible *positio* [referring to Duns Scotus, *Lectura I* d. 11 q. 2 n. 24, ed. Vat., vol. XVII, Civitas Vaticana, 1966, p. 136-137] agrees with that given by Ockham in whose *Summa logicae* we find what seems to be the last original treatment of the principles of impossible *positio* in the middle ages, (...). According to Ockham we may admit as *positum* in an impossible *positio* any impossibility which 'does not entail contradictories in virtue of rules and principles *per se*'. That is we may admit any impossible proposition which does not entail contradictories in a way which is 'manifestly clear to every understanding'". There are now several studies on the meaning of the rule 'ex impossibili sequitur quidlibet' in medieval logic; see, for example, J. Spruyt, Thirteenth-Century Positions on the Rule 'ex impossibili sequitur quidlibet', in: K. Jacobi (Hrsg.), *Argumentationstheorie. Scholastische Forschungen zu den logischen und semantischen Regeln korrekten Folgerns*, E. J. Brill, Leiden, 1993, p. 161-193.

<sup>22</sup> See S. Knuuttila, *Positio impossibilis* in Medieval Discussions of the Trinity, in: C. Marmo (ed.), op. cit., p. 282-285; Id., How Theological Problems Influenced the Development of Medieval Logic?, in: S. Caroti; R. Imbach; Z. Kaluza; G. Stabile; L. Sturlese (eds.), "Ad ingenii acultionem", Studies in Honour of Alfonso Maierù, FIDEM, Louvain-la-Neuve, 2006, p. 191-192. For further remarks on Scotus's uses of obligations rules see also Id., Modality as Alternativeness in John Duns Scotus, in: R. H. Pich (ed.), *New Essays on Metaphysics as scientia transcendens*, FIDEM, Louvain-la-Neuve, 2007, p. 145-157. The same point is emphasized by A. Vos, op. cit., p. 207-210.

<sup>23</sup> See again Ch. J. Martin, Impossible *positio* as the Foundation of Metaphysics or, Logic on the Scotist Plan?, op. cit., p. 258-261, as well as the references in notes 19 and 22.

'counterfactual' (or also 'counterpossible') analysis in which «intelligibility» is separated from "the idea of actualization in the real world"<sup>24</sup>. In the analysis before us, the first *positum* may be expressed as follows: (IP1) "A quantitatively infinite exists in actuality". This *positum* basically changes the modal operator of the Aristotelian account of the infinite, i.e., from "Potentially, there is an infinite in quantity" to "Actually, there is an infinite in quantity". To give a technically more precise formulation that is also consonant with Scotus's reading of Aristotle, we may change the formulation of this first step from "It is not possible, in actuality, that there is an infinite in quantity" (*verum naturaliter*) to "Actually, there is an infinite in quantity" (*impossibile naturaliter positum*)<sup>25</sup>. The logical character of this move from impossibility to actuality and hence possibility will be analyzed in the Conclusion.

**II.6** If an infinite being were to exist in actuality, then all parts of it *must be* simultaneously in actuality. The result of that imaginative reasoning, apparently possible at the conceptual level, is that all parts that were first actualized "in the infinite succession" (*in infinita successione*) and could only exist one after the other are now "conceived" (*conceptae*) to exist simultaneously *in actu*. Scotus seems to certify only the result of logical analysis; an actual infinite in quantity is possible to thought because it is non-contradictory, and it is within this process of conceiving it (which combines the infinite in quantity with actual existence) that (i) whole and (ii) perfect are incorporated within the intended content-construction of a given quantitative nature *X*. Perhaps it is more appropriate to say that the content-construction at this point is the simple result of an exercise of logical compatibility between different notions and two intuitions. After all, Scotus combines 'infinity' or 'unlimited' quantity of parts (a mathematical idea) with 'simultaneity' or 'actual simultaneity', and sees no contradiction in the 'actual infinity of quantity', *so long as* the notions of (i) whole and (ii) perfect can be viewed as contained in it. *Now* (i) the concept of *totum* is presupposed in that construction, because nothing else of that quantitative being stays outside itself. So too (ii) the concept of *perfectum*, because nothing is lacking in that being that could still be added to it in the category of exten-

<sup>24</sup> See S. Knuutila, *Positio impossibilis* in Medieval Discussions of the Trinity, in: C. Marmo (ed.), op. cit., p. 282f. In an original study, J.-L. Houdebine, *Excès de langages* (Hölderlin, Joyce, Duns Scot, Hopkins, Cantor, Sollers), Editions Denoël, Paris, 1984, p. 286f. (also note 3), suggests that "through the freedom of imagination" (as opposed to the "inhibition of imagination"), or through a certain "capacity of conceiving", Scotus effects a "transposition of a purely intellectual order", namely from the "indefinite successiveness" to the empirically non-realizable order of simultaneity of successive items, that is, the order of the quantitative infinite in actuality. This "intellectual imagination" would be shared by the discovery process of the 'transfinite' by Georg Cantor (p. 289-317); see also below.

<sup>25</sup> I have coined the expressions "*verum naturaliter*" and "*impossibile naturaliter positum*".

sion (*quantitas*)<sup>26</sup>. From (IP1) "A quantitatively infinite exists in actuality" two consequents follow, namely (C1) "Such an infinite in quantity is a whole thing" and (C2) "Such an infinite in quantity is a perfect thing".

**II.7** To be sure, it would be appropriate to offer a more formalized version of the argument. Let us call "P" the proposition "A quantitatively infinite exists in actuality" and "Q" the proposition "An actual infinite in quantity is a whole thing" (Consequent 1 above). Then we have:

1.  $\neg\Diamond P$  (*veritas naturaliter*);
2.  $P$  (*positio impossibilis naturaliter*);
3.  $P \wedge \neg Q$  (*Propositum* – denied because it is false; see II.2 and II.3);
4.  $\neg(P \wedge \neg Q)$  (*Propositum* – conceded as following from the denial of 3);
5.  $\neg\neg Q$  (*Propositum* – conceded as following from the admission of 2, and the concession of 4);
6.  $Q$ .

Then, by some subjunctive conditionalization (see the expression 'si posset ibi esse in actu' in II.3, and the determinations in VII. Conclusion), we arrive at

7. If it were to be that  $P$ , then it would be that  $Q$ .

The same would obtain if we had taken "Q" as the name of the proposition "An actual infinite being in quantity is a perfect thing" (Consequent 2 above)<sup>27</sup>.

### III. Imagination of an Actual Infinity in Entity

**III.8** It is only in the third step of his argumentation (see *Quodl.* 5 n. [3] 7-8) that Scotus comes to the actual infinity "in entity" (*in entitate*). What does it mean that Scotus speaks of 'entitas' here? Is doing so the same as to speak of a nature such as "deity" (*deitas*)? The answer seems to be that it is an indirect way of describing a real nature (namely, 'God') and a direct way of determining the real and most general concept of entity – a concept that everyone possesses. In both cases one would investigate and eventually obtain ontological infinity, but a presumably real infinite nature such

<sup>26</sup> See L. Honnefelder, *Scientia transcendens*, p. 110. See Duns Scotus, *Quaestiones quodlibetales* q. 5 n. [2] 6 p. 168: "Et omnes illae partes, quae in infinita successione essent reductae in actum et haberent esse post alias, tunc simul essent in actu conceptae; illud infinitum in actu vere esset totum, et vere perfectum totum, quia nihil sui extra; perfectum esset, quia nihil sibi deesset, imo nihil potest addi secundum rationem quantitatis, quia tunc posset excedi".

<sup>27</sup> I have taken his simple scheme (which derives from Walter Chatton) – but not the general conception or the particular purpose of the *positio* technique – from Ch. J. Martin, *Impossible positio as the Foundation of Metaphysics or, Logic on the Scotist Plan?*, op. cit., p. 267-268.



as God can only be approached indirectly through the direct logical construction of a real infinity in terms of entity. Since "entity" will be determined in *Quodl.* 5 n. [3-4] 7-11 through "infinity" (*infinitas*), just like "being" in *Ord.* I d. 2 p. 1 q. 1 n. 131-139 is determined through *infinitum*, here entity/*entitas* or "the character of being" of something is, just like the quidditative real concept of being/*ens*, the first transcendental – and not a more specific notion of either "reality" or the "intrinsic possibility" of real existence (as is the case in *Quodl.* 3 n. [2] 6-10)<sup>28</sup>. In this sense I see the use of *entitas* in *Quodl.* 5 n. [2-4] 5-11 as a more definite way of asking what a thing or a nature *is* on a *purely* transcendental level. It is the last formality obtained by the intellect in the analysis of knowledge – a purely determinable real concept, the most general and no further definable notion, and also a transcendental one because it is beyond any categoric determination and fully compatible with all pure perfections<sup>29</sup>. But significantly *entitas/ens* is at the same time the transcendental *quid* that can receive transcendental *qualia*<sup>30</sup> in terms of (a) 'properties of being' (*passiones entis*), (b) 'pure perfections' (*perfectiones simpliciter*), and (c) 'disjunct properties' or 'modes' (*passiones disiunctae*, in the sense of *each determination* of all transcendental disjunctions, which as *disiunctiones* – as *disjunctive* properties – are indifferently predicable of *ens*)<sup>31</sup>. As I shall try to prove – and this is why I offer this interpretation of the further conceptual supposition of the argument – it is implicit in *Quodl.* 5 n. [2-4] 5-11 that the discussion of 'infinity' in 'entity' can only make sense if understood as a way of articulating 'being', its 'properties', all pure perfections, and all modes of being that imply perfection *absolutely* (being then predicable possibly of only one 'quiddity' or 'being' like God's)<sup>32</sup>. Once that articulation is accomplished, *then* the logical *constructum* that shows specific transcendentals as compatible with one another can be ascribed as a proper real concept of a unique nature in the world that we call 'God'. This is not an instance of knowledge of the divine nature's existence but rather of the pure conceivability of the possibly most perfect content that metaphysics obtains of it.

**III.9** Two observations remain to be made. First, if we read carefully the structure of the argument, (A) we begin simply with infinity as unlimited continuation or separation of parts, (B) obtain then potential (mathematical) infinity, and (C) correct this idea of infinity with actual infinity in quantity. Only actual infinity is legitimate infinity, and only an actual

<sup>28</sup> See, for instance, L. Honnefelder, *Duns Scotus*, p. 75f.

<sup>29</sup> Id. *ibid.*, p. 56-59. See also below.

<sup>30</sup> I.e., determinations predicated 'in quale'.

<sup>31</sup> See Duns Scotus, *Ordinatio* I d. 3 p. 1 q. 3 n. 131, ed. Vat., vol. III, Civitas Vaticana, 1954, p. 81; *Ordinatio* I d. 8 p. 1 q. 3 n. 113-115, ed. Vat., vol. IV, Civitas Vaticana, 1956, p. 205-207; *De primo principio* IV concl. 3 n. 53-54 p. 64-66; *Quaestiones quodlibetales* q. 1 n. [8] 19-23 p. 14-17.

<sup>32</sup> Further in *Quaestiones quodlibetales* q. 5 n. [7-12] 17-29 p. 168-182, Duns Scotus shows that there cannot be several beings that are distinct and formally infinite.

infinity can be an infinity in entity – whatever that might be. As a parallel conclusion of the argument, we shall see that only actual infinity in entity can be legitimate infinity. Second, it is possible to say that Scotus brings further suppositions into the *positio*. After all, if he has (A) the (Aristotelian) supposition of unlimited continuation or separation of parts for infinity, which serves for quantity, he has also the supposition of illimitation as such – later in *Quodl.* 5 n. [4] 11, with reference to John Damascene, Scotus twice uses “*interminatam*” practically as a synonym for “*infinitem*”<sup>33</sup>. It is on account of that intuition that we have had already offered critical remarks on the notions of ‘whole’ and ‘perfect’. I think that Scotus here realizes that he must consider the intuition of illimitation in a more general and strict sense, and that demands a careful revision of the concept of ‘whole’ and above all, as we shall see, of the concept of ‘perfect’<sup>34</sup>.

**III.10** Let us continue our outline of the argument. Scotus next argues, in a way that seems almost rash, that the infinite in entity must be understood “proportionally” or “analogically” (*proportionabiliter*) to that actual infinity imaginatively built in the category of quantity. Within such a ‘proportional’ or ‘analogical’ understanding, one should think of a reality or a being that cannot be surpassed *in entity* through anything at all, and that will finally fulfill the requirement of analytic inclusion of the concepts of whole and perfect<sup>35</sup>. It is important to realize that, precisely because of this, such a proportional understanding is itself part of the concept formation process *secundum imaginationem*. I propose that we have in this third step a second *positum* (IP2): “An infinite in entity does exist in actuality”. And it is possible to interpret the need for revising those suppositions above as a consequence of it. Here too, the starting point for the application of *positio impossibilis* admits a different, perhaps technically more precise formulation; the proposal is to replace “It is not possible, in actuality, that there is an infinite being” (*verum naturaliter*) with “Actually, there is an infinite in entity” (*impossibile naturaliter positum*). The logical character of the move from impossibility to actuality remains to be analyzed (see Conclusion).

**III.11** The formation of the concept of infinite being is clearly not yet finished; the formal criteria of the infinite in entity can only be met if the imperfection that still adheres *unavoidably* to the infinite in quantity is left

<sup>33</sup> ‘Infinitem’ and ‘illimitatum’ appear as synonyms also in *Quaestiones quodlibetales* q. 6 n. [6] 15 pp. 210-211, where also the expression ‘extensive illimitatio’ is to be found (it stands in contrast there to ‘intensive illimitatio’).

<sup>34</sup> As we notice from *Physica* III 6 207a8-15, Aristotle understands ‘whole’ (*holon*) and/or ‘perfect’ (*teleion*) as analytic contents of the ‘infinite’ (*apeiron*) too.

<sup>35</sup> See Duns Scotus, *Quaestiones quodlibetales* q. 5 n. [3] n. 7 p. 168: “Ex hoc ultra: Si in entibus intelligamus aliquid infinitum in entitate in actu, illud debet intelligi proportionabiliter quantitati imaginatae infinitae in actu, sic ut ens illud dicatur infinitum quod non potest ab aliquo in entitate excedi, et ipsum vere habebit rationem totius et perfecti”.

behind. In other words, if the actual infinity in quantity, which includes (i) the concept of whole and (ii) the concept of perfect, is applied without proportionality to the idea of actual infinity in entity, 'whole' and 'perfect' would remain enough for infinity *in quantity*, however the quantitative 'perfect' would not be enough for infinity *in entity*. Although actual infinity in quantity is *complete* in this category, since it possesses all its parts *in actu* and nothing is left out of *that* perfect whole, each part of the quantitative whole stays out of each other part of the same whole and, therefore, *each part* of the quantitative whole is not perfect in itself. Grounded on this description, it is possible to affirm that the quantitative whole consists always of parts really different from each other and that are as such limited, hence, imperfect<sup>36</sup>. And it seems also manifest that the sense of (ii) 'perfect' used to this point was that of 'complete'<sup>37</sup>, where 'completeness' can be characterized in such a way as to be compatible with 'limitation' (completeness in extension or quantitative parts) and with 'illimitation' too (necessarily then completeness in entity or ontological parts).

**III.12** By contrast, an actual infinite in entity is something that has nothing of entity outside itself, because its "totality" (*totalitas*) does not depend on parts (*ex aliquibus*) that are imperfect in entity. Scotus clearly envisages an idea of actual infinity that can fulfill a *strict* concept of perfect. There is no doubt that *a whole* of specific kind or a 'whole of something', of quantity, of entity, etc., in order to be "a totally whole" (*totaliter totum*), cannot have any extrinsic part, and for this reason, as already indicated, there is a relative sense in which an actual infinite in the (mere) category of quantity – or a whole of quantity – is *perfect* too<sup>38</sup>. The reason seems to be that whatever is an actual qualified whole or an actual whole of *something* intrinsically limited – of numerical quantity, of quantity of matter, hence of numerical parts, of parts of a body, etc., – implies being, in a qualified sense, perfect or even 'complete'<sup>39</sup> in something intrinsically limited (in

<sup>36</sup> See Duns Scotus, *Quaestiones quodlibetales* q. 5 n. [3] n. 7 p. 168: "Totius quidem, quia licet totum infinitum actu in quantitate nulla parte sui nec etiam parte quantitatis talis careret, tamen quaelibet pars esset extra aliam, et sic totum esset ex imperfectis".

<sup>37</sup> "Complete" is also a possible translation of the Latin "*perfectum*".

<sup>38</sup> See Duns Scotus, *Quaestiones quodlibetales* q. 5 n. [3] n. 7 p. 168-169: "Sed ens infinitum in entitate sic nihil entitatis habet extra, quia nec eius totalitas dependet ex aliquibus imperfectis in entitate; sic enim totum est quod nullam habet partem extrinsecam, quia tunc non esset totaliter totum; ita etiam quamvis infinitum in actu esset perfectum in quantitate, quia sibi secundum se totum nihil quantitatis talis deesset, (...)". In what concerns the Aristotelian definition of the infinite in *Physica* 207a7-14, see the remark of W. D. Ross, *Commentary*, op. cit., p. 368: "The infinite is that of which some part is always beyond; that of which there is nothing beyond is complete and whole. 'Complete' and 'whole' mean very much the same thing; (...)". See also notes 13, 34, 37.

<sup>39</sup> Again (see also note 37), in relation to quantity, this seems to be a quite reasonable translation for 'perfectum'; to be complete can be an intrinsic perfection of a quantitative whole, but it cannot be an intrinsic perfection of any part of that same whole. This second aspect – or rather this kind of whole that would evade this very difficulty – seems to be what Scotus has in mind when he thinks of a strict notion of infinity.

numerical parts, in parts of a body, etc.)<sup>40</sup>. But in this case it is easy to see, as a consequence, that an actual quantitative infinity has intrinsic limitations; whatever includes internal things/parts intrinsically limited cannot but be intrinsically limited too. This 'intrinsic limitation' (my expression) seems to be what Scotus has in mind when he states that any part of a quantitative infinity *lacks* something regarding quantity itself, namely, something that would be in another part of the same whole. The respect for 'proportionalities' – or the 'analogical' sense of the argument – in the passage from actual infinity in quantity to actual infinity in entity here effectively conveys the following advice: the imperfection or the intrinsic limitation of each part of the actual *quantitative* infinite cannot be removed<sup>41</sup>. And because an actual quantitative infinite or a 'whole' of quantity is compatible with 'limited completeness', 'whole' and 'perfect/complete' are not synonymous and the 'limited completeness' of a 'whole' of quantity does not constitute actual infinity.

**III.13** The limitations of an account of actual infinity *in quantity* recognized above demand a remark on the structure of the argument. Exploring it only at the beginning of the *positio*, Scotus nowhere explicitly denies the very idea of an actual infinite in quantity. That being said, it is nonetheless clear that what would make it conceivable cannot be satisfied by it in its own terms. Accordingly, we may think that he would propose something like the following: Let us call "P" the proposition "An infinite in quantity exists in actuality" and "R" the proposition "An actual infinite is a strict whole and a strict perfect". Then we would have:

1.  $\neg \Diamond P$  (*veritas naturaliter*);
2.  $P$  (*positio impossibilis naturaliter*);
3.  $R$  (*Propositum* – conceded because it is true; see III.7-12);
4.  $P \wedge R$  (*Propositum* – denied because it is false; see III.7-12);
5.  $\neg (P \wedge R)$  (*Propositum* – conceded as following from the denial of 4);
6.  $\neg R$  (*Propositum* – conceded as following from the admission of 2, and the concessions of 3 and 4).

<sup>40</sup> I think that this could be easily exemplified in the "complete" series of (cardinal) natural numbers in their order of magnitude (1,2,3,4,5...), where each prior number does not contain the posterior one. Although such a series can be an example of quantitative infinity and illustrates the point about imperfect parts and imperfect whole, it cannot be an example of quantitative illimitation, since that series has no end, although it does have a beginning.

<sup>41</sup> See Duns Scotus, *Quaestiones quodlibetales* q. 5 n. [3] n. 7 p. 169: "(...), tamen cuilibet parti eius deesset aliquid quantitatis, quae scilicet esset in altera, nec ipsum esset sic perfectum, nisi quodlibet eius esset imperfectum". Notice here that the thought of an actual infinite magnitude in extension is of interest for Scotus only insofar as he sees in it a necessary step for constructing the idea of an infinite nature in intensity. G. Sondag suggests a comparable move in B. Bolzano (in his *Paradoxen des Unendlichen*), who requires a similar step from infinite quantity to infinite "quality"; cfr. G. Sondag, *op. cit.*, 118-119, also notes 27 and 28.

Then, through some subjunctive conditionalization (see also the expression 'si esset possibilis' in *Quodl.* 5 n. [3] 8 and under III.16, as well as the determinations in VII. Conclusion), we would arrive at

7. If it were to be that *R*, then it would be that not *P*.

But the difficulty with this approach is that it would be at odds with the rules of the logical game 'de obligationibus'. After all, that game follows the deontic-logical principle that the *respondens* has the duty of granting the *positum* as well as granting and considering propositions that are connected with it in some way (i.e., *pertinens sequens*, *pertinens repugnans*, *indifferentes*, etc.)<sup>42</sup>. In light of this, it seems to me that it is much easier and fairer to *Quodl.* 5 just to say that, in order to obtain the concept of infinite being, and for specific reasons concerning its contents (see III.7-12), Scotus begins a new obligational game in (IP2)<sup>43</sup>.

**III.14** Differing from an actual quantitative infinite, the infinite being *is perfect* in such a way that: something is lacking in entity neither in the actual infinite whole (*nec sibi*) nor in any part of this same whole (*nec alicui eius*), (ii\*) it is perfect or 'complete' in entity, or it contains at once and actually everything of entity as such, and (ii\*\*\*) the parts of its perfection or completeness have no intrinsic limitation. And whatever includes internal things/parts intrinsically unlimited cannot but be intrinsically unlimited too. This 'intrinsic illimitation' (again, my expression) seems to be what Scotus has in mind when he states that any part of an ontological infinity lacks *nothing* regarding entity as such, namely, something that would be in another part of the same whole<sup>44</sup>. The respect for 'proportionalities', in the passage from actual infinity in quantity to actual infinity in entity, amounts to the following conceptual construction: it is possible to think of an actual infinite whole that is perfect or complete in that of which it is a whole (in entity as such), and whose parts have no intrinsic limitation in that of which they are parts. And if it follows now from (IP2) "An infinite in entity exists in actuality", that both (C1) "Such an infinite in entity is a whole with no intrinsic limitation" and (C2) "Such an infinite in entity is a perfect in the strictest sense", then it is clear that I have labored to make precise the content of the supposition of 'illimitation' or, in other words, of the suppositions 'whole' and 'perfect', because the very conceivability of Scotus's second *positio impossibilis* requires this movement.

**III.15** Now, let us call "P" the proposition "An infinite in entity exists in actuality" and "Q<sub>1</sub>" the proposition "An actual infinite in entity is a strict whole" (Consequent 1 above). Then we have:

<sup>42</sup> See the references in notes 19-23.

<sup>43</sup> See more on that in VII. Conclusion.

<sup>44</sup> See Duns Scotus, *Quaestiones quodlibetales* q. 5 n. [3] n. 7 p. 169: "Sed ens infinitum sic est perfectum, quod nec sibi nec alicui eius deest aliquid".

1.  $\neg\Diamond P$  (*veritas naturaliter*);
2.  $P$  (*positio impossibilis naturaliter*);
3.  $P \wedge \neg Q_1$  (*Propositum* – denied because it is false; see III.7-14);
4.  $\neg(P \wedge \neg Q_1)$  (*Propositum* – conceded as following from the denial of 3);
5.  $\neg\neg Q_1$  (*Propositum* – conceded as following from the admission of 2, and the concession of 4);
6.  $Q_1$ .

Then, through some subjunctive conditionalization (see the expression 'si esset possibilis' in *Quodl.* 5 n. [3] 8 and in III.16, as well as the determinations in VII. Conclusion), we arrive at

7. If it were to be that  $P$ , then it would be that  $Q_1$ .

Again, the same would obtain if we had taken " $Q_2$ " as the name of the proposition "An actual infinite in entity is a strict perfect" (Consequent 2 above).

**III.16** In order to understand Scotus's intentions in the present attempt at concept formation, it is crucial to pay attention to the summary he makes of the three general steps taken thus far. A final feature of the summary has been surprisingly neglected in the *Scotusforschung* or has at least gone unnoticed: (1) the Subtle Doctor begins with the definition of the potential infinite according to Aristotle's *Physica*, (2) turns it *secundum imaginationem* into the notion of actual infinity in quantity "if that were possible" (*si esset possibilis*), and (3) finally applies it – *secundum imaginationem*, we may infer – to actual infinity in entity "where it is possible" (*ubi est possibilis*) (!)<sup>45</sup>. Scotus changes the language of subjunctive conditional possibility in the second step – where he showed what had to follow from (IP1), once it was conceded – to language of indicative actual possibility in the third step. What is the significance of this change? If we look at Scotus's words, it certainly means that it is in terms of 'entity' or 'being' that a concept of actual infinity is ultimately *possible* and hence something positively *conceivable*. It is an *ontological* infinity that is ultimately a *conceivable* infinity, since infinity is that which fulfills strictly (i) the concept of whole and (ii) the concept of perfect, where 'strict whole' and 'strict perfect' mean respectively total absence of extrinsic parts and of intrinsic limitation. And given that 'whole' and 'perfect' are the essential structural parts of the concept of infinity that Scotus envisages, I do think that important (speculative) remarks about them still need to be made.

**III.17** A (i) strict concept of whole signifies an absence of extrinsic parts; if some  $X$  is a whole, then it is a reality of which it is not possible to

<sup>45</sup> See Duns Scotus, *Quaestiones quodlibetales* q. 5 n. [3] n. 8 p. 169: "Sic ergo, ex ratione infiniti posita in III *Physicorum*, 1, applicando secundum imaginationem ad infinitatem actualem in quantitate, si esset possibilis, ulterius applicando ad infinitatem actualem in entitate, ubi est possibilis, (...)".

say that 'it lacks something'. However, at least as far as Scotus is concerned, 'whole' is not as such an unspecifiable concept – it is a concept and a term that admits or even demands complement. Moreover, when taken as a substantive, it is a kind of collective term as well, for it suggests discernable parts. So it must be said that every whole is the whole 'of something' and that it has some kind of 'parts'. In this case, even if 'actual infinite whole' expresses part of the content of the concept of (ontological) infinite, it cannot express all of it, because (ii\*) even if an actual infinite whole implies completeness – and also the idea of 'perfection' in terms of 'completeness'<sup>46</sup> – it does not imply as such a 'strict idea of perfect'; (ii\*\*\*) the strict idea of perfect seems to imply completeness of infinite parts that have no intrinsic limitation. And that is the reason why the strict notion of infinity, the ontological infinity, obtains only when thought goes beyond merely *quantitative* parts to 'parts' of *entity* – such parts are 'that of which' infinity or the actual whole now specifically consists. The strict notion of infinity, containing (i) strict whole and (ii) strict perfect, is at the same time synonymous with ontological infinity (infinity in entity), and it can only be (it is only conceivable) as ontological infinity. Because 'entity' is the only thing for which we can speak of perfection analytically in terms of 'completeness' (which is symmetric to 'whole of something' and 'totality of parts') and 'absence of intrinsic limitation' (see below), having completed steps (1) and (2), we conclude that (3) 'infinity in entity' is *the very way* of conceiving 'actual infinity'. As I will try to show in greater detail, while 'strict whole of something in totality of parts' is a notion easier to conceive, 'strict perfect' is a more complicated notion. That its very content (thus far) is 'intrinsic illimitation' or, put differently, 'intrinsic illimitation in entity' (note: both expressions are my own), this is the conceptual step with which Scotus concludes the first division of this case of concept formation: through the conceivable actual infinity in entity (1, 2, and 3), "we can have some sort of understanding [*aliqualem intellectum*] of how a being intensively infinite [*ens infinitum intensive*] in perfection [*sive in perfectione*] or in virtue [*vel in virtute*] must be conceded [*concedendum est*]"<sup>47</sup>. It is interesting to note

<sup>46</sup> See also above under III.12 and III.14. See also note 39.

<sup>47</sup> See Duns Scotus, *Quaestiones quodlibetales* q. 5 n. 13| n. 8 p. 169: "(...) habemus aliqualem intellectum qualiter concedendum est ens infinitum intensive, sive in perfectione vel in virtute". G. Sondag, *Jean Duns Scot sur l'infini extensif et l'infini intensif*, op. cit., p. 120, note 35, observes correctly that Aristotle also admits, just as Scotus does in the second step of his argumentation, that if an [actual] quantitative infinite were possible, then it would be a whole. Nevertheless, the possibility of an actual infinite in entity and/or of perfection of nature – the very theme of the third step of Scotus's argumentation – is definitely denied by Aristotle in *Physica* I 2, 185b1-5 (in terms of an infinite *ousia*, of a thing/substance infinite, and in terms of an infinite *poion*, that is, of an infinite mode/kind or of an infinite quality). See Aristotle, *Physica*, translated by R. P. Hardie and R. K. Gaye, op. cit., *Physica* I 2, 185b1-5: "For to define the infinite you must use quantity in your formula, but not substance or quality. If then Being is both substance and quantity, it is two, not one: if only substance, it is not infinite and has no magnitude; for to have that it will have to be a quantity". See also Conclusion (VII.37ff.).

that the verb ‘concedo’ is used here, since this is the verb used within the technical language of obligational games, that is, the verb indicating the acceptance by a *respondens* of what is proposed by an *opponens* as being a valid consequence derived from a *positum* or a *positum* plus a given set of further propositions. In the case at hand, ‘concedo’ expresses more specifically what must follow as affirmations of the conceivability of a *positum* such as (IP2).

**III.18** The summary above is very important for what comes next. After all, it is a good question what should be taken as ‘parts’ of an actual infinity in entity. If we understand what numerical parts are, what are parts of entity? More importantly, how can such a discourse on parts avoid real composition through limited parts? In his *first* approach<sup>48</sup> to the idea of “parts” of an infinite being in *Quodlibet* 5, Scotus states vaguely that an actual infinity in entity has ‘perfections’ or ‘virtues’ as its ‘parts’ – more exactly, as its kinds of parts. Here a dissimilarity between steps (2) and (3) becomes more apparent: How is the ‘proportionality’ or ‘analogy’ between actual infinity in quantity or numerical parts to be applied to actual infinity in entity? The actual infinite in quantity has *numerically* infinite parts. Scotus does not affirm – and also does not deny – that the actual infinite in entity has numerically infinite parts (‘perfections’, ‘virtues’, etc.). His concern seems rather to be that we now change the merely arithmetical-mathematical account of infinity into a notion of ‘intensity’: an infinite intensively is a being whose *intensity* – not *number* (!) – in ‘entity’ or ‘beingness’ is infinite. In this sense, ‘infinite’ does not concern primarily the number of parts, but primarily their (degree of) intensity in entity. The ‘intensity’, not the number, of the parts of entity and the parts capable of a (degree of) intensity in entity that is ‘infinite’ seem to be the two ideas able to explain *intrinsic illimitation* or *absence of intrinsic limitation* of whole and parts – the perfection of the whole and of the parts. Both ideas are covered *respectively* by what I call, in the next step of the technique, a “modal description” (intensity at the degree of infinity) and a “quidditative description” (particular whole and parts) of *ens infinitum*.

#### IV. Two Descriptions

**IV.19** It is interesting to note that, after having proposed a possible concept (*ratio, intellectus*) of actual infinity in entity or of an infinite being in entity<sup>49</sup>, Scotus states in *Quodl.* 5 n. [4] 9 that he is now able to offer a description of that reality, and he offers it in two different moments (“ex hoc, possumus ens ‘infinitum in entitate’, sic describere...”; “Potest etiam

<sup>48</sup> The second approach to the idea of parts of an infinite being is briefly examined below under “IV. Two Descriptions”, namely in IV.27 and IV.28.

<sup>49</sup> “Actual infinity in entity” and “infinite being in entity” are just the same concept.



describi..."). If we understand steps (1), (2), and (3) above as ways of establishing or constructing a complex concept (actual infinity in entity) in terms of strict conceivability of its contents (whole with no intrinsic limitation and a perfect thing in the strictest sense) – so that we propose them as its *analytic* contents and, in a sense, the structure or the necessary and sufficient conditions of its definition – how should a “description” be understood? I think it should be understood as (a) a more determinate way of providing component (quidditative) contents to such a reality and concept, and as (b) a more descriptive way of specifying the modal aspect of those component (quidditative) contents. In the end, we shall see that the content of “actual infinity in entity” or “actual infinite in entity” – whose structure of conceivability was offered in (1), (2), and (3) – is the result of combining what I shall call (I) “quidditative description”<sup>50</sup> and (II) “modal description”. If it should be asked in what sense this descriptive phase of the argument is related to the *positio impossibilis* phase of it, I suggest that in this case once again the first phase provides the structural steps for the conceivability of the intended concept of the ontological infinite, and the second phase operates with *the very contents* that can satisfy those structural steps, as well as with *the right idea of their “dimension”*. In this sense, “structural conceivability” and “contents and modes of conceivability” respectively belong altogether to the general strategy of concept formation.

**IV.20** In the (I) “quidditative description”, the “being infinite in entity” is described “as a being that lacks no entity in the way that one single being is able to possess it [i.e., the totality of entity]”. Surely this definition reminds us of the construction of the concept of infinite being in terms of strict whole and strict perfect. But Scotus adds clearly the clause “in the way ... is able to” (*eo modo quo possibile est*), that is, a clause of possibility for a single being to possess entity as a whole; presumably there is a way in which this is possible and conceivable, since such a single being cannot possess “in itself” (*in se*), “really and formally” (*realiter et formaliter*), “every entity whatsoever” (*omnem entitatem*) “through identity” (*per identitatem*)<sup>51</sup>. A short exploration of this subject will show that Scotus confirms here what, in terms of quidditative content, actual infinity, as strict whole and strict perfect in entity, is supposed to be. This seems to mean that such a

<sup>50</sup> I am using “quidditative” because of the use of ‘quiditas’ for essence and also for a formally distinguished content that belongs to an essence on the basis of what the essence is in respect of its mode; the expression “essential description” would work as well. Concerning what belongs or expresses formally the properties of the divine essence Scotus uses ‘quiditas’ for *perfectiones simpliciter* in *Ordinatio* I d. 8 p. 1 q. 4 n. 220 p. 275. Of course, the expression “quidditative” here does not concern the discussion of the nature and predicative character of transcendentals called ‘perfectiones simpliciter’. Surely perfections called ‘quiditates’ do not express formally the divine essence.

<sup>51</sup> See Duns Scotus, *Quaestiones quodlibetales* q. 5 n. [4] 9 p. 169: “Ex hoc, possumus ens ‘infinitem in entitate’, sic describere, quod ‘ipsum est cui nihil entitatis deest, eo modo quo possibile est illud haberi in aliquo uno’; et hoc pro tanto additur, quia non potest in se realiter et formaliter per identitatem omnem entitatem habere”.

being or nature – which human intellect does not know as such – according to the whole and to the perfect both possesses only what is strictly of ontological perfection (quidditatively, that which has no intrinsic limitation) and properties and perfections as well that allow some kind of real *and* formal distinction – or, using the official expression for such a distinction, some “formal non-identity based on the real thing”<sup>52</sup>. At the same time, it seems correct to affirm that the concepts of ‘whole’ (of actual totality of parts) and of ‘perfect’ (what lacks nothing as such), belonging analytically to the concept of the actual infinite that is hence structurally conceivable as *perfect whole*, are describable *in ontological terms* so long as they concern ‘entity’, namely not ‘every entity’, but fundamentally that ‘entity’ and further quidditative aspects of entity *with no intrinsic limitation* that can be possessed by a single being. Accordingly, within the first approach to the notion of “parts” of a *perfect whole* or an actual infinity in entity, we shall *exclude from it* any nature and any properties that in their full conception<sup>53</sup> contain the notion of finitude, such as “human being”, “rational animal”, “laughable”, “dog”, “three”, “stone”, etc. And we must definitely *include in such an essence*, through real identity, all “pure perfections” (*perfectiones simpliciter*)<sup>54</sup>, comprising as well all divine attributes – i.e., attributes exclusive of God – under the expression “divine perfections” (*perfectiones divinae*)<sup>55</sup>. The fact that every pure perfection is compatible with the divine/infinite essence through real identity and stays in conformity with its sim-

<sup>52</sup> See Duns Scotus, *Ordinatio* I d. 2 p. 2, q. 1-4 n. 390. 404 p. 350. 357. As is well known, both real identity and formal non-identity *a parte rei* are “real” independently of the intellect. *X* and *Y* would be formally identical iff either *X* were part of the formal reason of *Y*, or *Y* were part of the formal reason of *X*. Where this is not the case, and *X* and *Y* are really the same, *X* and *Y* are formally non-identical.

<sup>53</sup> The one that includes essential aspect plus the intrinsic mode of it according to the *prima divisio* “infinite”-“finite”; see L. Honnefelder, *Duns Scotus*, p. 88-94; R. Cross, *Duns Scotus on God*, p. 114; Duns Scotus, *Ordinatio* I d. 8 p. 1 q. 2 n. 32-38 p. 165-168. On the concept of an essence plus its intrinsic mode as a “*conceptus per se*” see Duns Scotus, *Ordinatio* I d. 3 p. 1 q. 1-2 n. 58 p. 40.

<sup>54</sup> Cfr. Duns Scotus, *Quaestiones Quodlibetales* q. 1 n. [4] 8-10 pp. 9-10. Cfr. also G. Sondag, Jean de Damas et Jean Duns Scot sur l’infinité de l’essence divine, in: *Chora – Revue d’Études Anciennes et Médiévales*, 3-4 (2005-2006), p. 314-315. The following are certainly sufficient characterizations of pure perfections such as “wisdom”, “goodness”, “life”, etc.: they are defined in contrast to the positive thing with which they are incompatible; they make their possessor better absolutely speaking; they are compossible with one another; each of them is irreducibly simple; each of them is incompatible with finity as such; they are all equally perfect; none of them is formally incommunicable; they are all transcendentals; they apply to things independently of their types; they are not simply coextensive with being; they are not simply proper to God alone. Cfr. J. Seifert, A Vontade como perfeição pura e a nova concepção não-eudaimonística de amor segundo Duns Scotus, in: *Veritas* 50:3 (2005), p. 51-78.

<sup>55</sup> See Duns Scotus, *Quaestiones quodlibetales* q. 5 n. [24-25] 55 p. 196: “Et cum additur quod infinitas intellectus est propter identitatem eius ad essentiam, respondeo: primum omnino in divinis, ut tactum est in prima quaestione, est essentia ut essentia, quae, secundum Damascenum, est pelagus propter comprehensionem omnium perfectionum divinarum; ista est infinita non tantum intensive in se, sed etiam virtualiter

plicity can be seen in the finely worked out definition of the *perfectiones* in *Quodl.* 5 n. [13-14] 30-32, where Scotus even calls the “infinite intensively” (*infinitum intensive*) “a pure perfection”<sup>56</sup> because of its absolute illimitation. This seems to be the sense in which such a being does not possess “in itself”, “really and formally” through some “identity”, “every entity whatsoever”. It can possess ‘in itself’ *really* through identity – even if *formally* with no identity – quidditative aspects of entity *with no intrinsic limitation*<sup>57</sup>. And precisely those contents that can fulfil this description are then the quidditative contents – contents relative to ‘entity’ – of the infinite being that have to be examined and incorporated into the process of concept formation<sup>58</sup>. Scotus is presumably saying that several quidditative contents cannot ever be assumed as belonging to the infinite being or nature. Therefore, from (IP2) “An infinite in entity exists in actuality” (or, assuming what was concluded before in (C1) and (C2), “A perfect whole of being exists”) it follows (C3) “There is a nature that, together with its essential aspects or perfections really identical and formally different, has *no intrinsic limitation*, and it can be possessed by a single being”.

**IV.21** Let us call “P” the proposition “An infinite in entity exists in actuality” and “Q<sub>3</sub>” the proposition “There is a nature that, together with its essential aspects or perfections really identical and formally different, has *no intrinsic limitation*, and it can be possessed by a single being” (Consequent 3 above). Then we have:

1.  $\neg\Diamond P$  (*veritas naturaliter*);
2.  $P$  (*positio impossibilis naturaliter*);
3.  $P \wedge \neg Q_3$  (*Propositum* – denied because it is false; see III.6, III.14, and IV.20);
4.  $\neg(P \wedge \neg Q_3)$  (*Propositum* – conceded as following from the denial of 3);

primo et per se continens omnia intrinseca; (...). See also Duns Scotus, *De primo principio* IV concl. 3 n. 53 p. 64: “TERTIA CONCLUSIO: *Omnis perfectio simpliciter, et in summo, inest necessario naturae summae*. Perfectio simpliciter dicitur quae in quolibet est melius ipsum quam non ipsum”.

<sup>56</sup> See Duns Scotus, *Quaestiones quodlibetales* q. 5 n. [13] 30 p. 182: “Omnis perfectio simpliciter est communicabilis; omne infinitum intensive est perfectio simpliciter; ergo etc.”. See id *ibid.*, n. [13] 31 p. 183: “Probatio maioris primae: Quia secundum quod colligitur ex intentione Anselmi, *Monologio* 15, perfectio simpliciter est quae in quolibet habente ipsam melius est ipsam habere, quam non-ipsam habere”; *ibid.*: “(...); sed intelligitur ibi ‘non-ipsam’ pro quocumque sibi impossibili etiam positive, ut sit sensus: perfectio simpliciter est in quolibet melior quocumque sibi impossibili”.

<sup>57</sup> Surely Scotus is conscious in *Quodlibet* 5, as he is in *Ordinatio* I d. 8 p. 1 q. 1-2 n. 1-38 p. 153-168, that real distinction of attributes in an entity or essence implies composition, and composition implies limitation, viz. potentiality. In this sense, one can arguably infer that “infinity” explains real and essential “simplicity”; see the excellent account of simplicity in R. Cross, *Duns Scotus on God*, p. 99-114, 112-113 (Chapter 6: “Divine Simplicity”).

<sup>58</sup> Even if Scotus does not develop the issue here, I think that such contents must be thought of in terms of pure perfections.

5.  $\neg Q_3$  (*Propositum* – conceded as following from the admission of 2, and the concession of 4);
6.  $Q_3$ .

Then, by strict (necessary) implication (see the expression ‘ubi est possibilis’ in *Quodl.* 5 n. [3] 8 and under III.16, as well as the determinations in VII. Conclusion), we arrive at

7. If it is that  $P$ , then it is that  $Q_3$ .

**IV.22** In the (II) “modal description”, Scotus, even if he does not use the expression, actually begins the analysis of the infinite in terms of the “transfinite”, a term that seems to signify “that which is beyond all finite, however great this may be”<sup>59</sup>. In this description “through excess” (*per excessum*), the being infinite in entity is that “which exceeds any finite being whatsoever not according to some limited degree [*determinatam proportionem*] but [in a degree] beyond every determinate or determinable degree”<sup>60</sup>. I assume that Scotus “describes” further – as specified in IV.19 – a being that is infinite in entity, i.e., in its character of being or nature. Now ontological infinity is described firstly in terms of «proportion» or «degree» (*proportio*) – once again not in terms of number in ‘entity’. It seems right to say that ontological infinity is described in terms of “proportion” or “degree” (*proportio*) of “intensity” (*intensio*)<sup>61</sup> in “entity” (*entitas*). Scotus ex-

<sup>59</sup> See G. Sondag, *Jean Duns Scot sur l’infini extensif et l’infini intensif*, op. cit., p. 116 (115-121). A comparison between Scotus’s “infinite” and G. Cantor’s “transfinite” in mathematics is beyond the scope of this essay. See on this J. P. de Tudela y Velasco, *Identidad, forma y diferencia en la obra de Juan Duns Scotus. Una aproximación matemática al problema de su interpretación*, Editorial de la Universidad Complutense, Madrid, 1981, p. 117-150. On transfinite methods and actual infinity in the philosophy of mathematics, see the remarks in S. Körner, *The Philosophy of Mathematics*, Dover Publications, New York, 1986 (republication of the second edition, 1968), p. 60ff., 111ff.

<sup>60</sup> See Duns Scotus, *Quaestiones quodlibetales* q. 5 n. [4] 9 p. 169: “Potest etiam describi per excessum ad quodcumque aliud ens finitum sic: ‘Ens infinitum est quod excedit quodcumque ens finitum, non secundum aliquam determinatam proportionem, sed ultra determinatam proportionem vel determinabilem’”. See also Duns Scotus, *Ordinatio* I d. 2 p. 1 q. 1-2 n. 132 p. 206-207: “Minor probatur, quia infinitum non repugnat enti; sed omni finito maius est infinitum. Ad istud aliter arguitur, et est idem: cui non repugnat infinitum esse intensive, illud non est summe perfectum nisi sit infinitum, quia si est finitum potest excedi vel excelli, quia infinitum esse sibi non repugnat; enti non repugnat infinitas; ergo perfectissimum ens est infinitum”. The Scotist definition will have strong influence on the “infinetists” of the 14<sup>th</sup>-century, which, similarly, would try to construct a non-Aristotelian concept of the actual infinite magnitude (John of Bassolis, Francis of Mayronis and, above all, Gregory of Rimini). See G. Sondag, *Jean Duns Scot sur l’infini extensif et l’infini intensif*, op. cit., p. 116-117.

<sup>61</sup> The noun ‘intensio’ does not appear in *Quodl.* 5 n. [2-4] 5-11, but rather ‘infinitas intensiva’ or ‘infinitum intensive’, where ‘intensive’ (see *Quodl.* 5 n. [4] 10) is clearly an adverb. On the use of ‘intensio’ see note 78. In any case, it is fair to say that ‘infinitas’ of ‘entitas’ or ontological infinity for Scotus can be analogically “measured” by a scale of intensity – entity can be analogically measured by a scale of intensity – and that intensity is able to admit a kind of degree. Scotus is searching for a language that is able

plains here his basic thesis that the being that is actually infinite – that is analytically a strict whole and a strict perfect in entity – is or has intensive infinity; infinity is *an intensity of entity*, namely, *in that being that is a strict whole and a strict perfect in entity*. This way of considering infinity is the only way to make sense of the idea of actual infinity, hence of ontological actual infinity. So from (IP2) “An infinite in entity exists in actuality”, (or assuming what was obtained in (C1), (C2), and (C3), “A perfect whole of being, with no intrinsic limitation, exists”), follows (C4) “There is a being whose degree of intensity in entity is infinite or whose intensity in entity is infinity”.

**IV.23** Let us call “P” the proposition “An infinite in entity exists in actuality” and “Q<sub>4</sub>” the proposition “There is a being whose degree of intensity in entity is infinite or whose intensity in entity is infinity” (Consequent 4 above). Then we have:

1.  $\neg\Diamond P$  (*veritas naturaliter*);
2.  $P$  (*positio impossibilis naturaliter*);
3.  $P \wedge \neg Q_4$  (*Propositum* – denied because it is false; see III.6, III.14, and IV.20-22);
4.  $\neg(P \wedge \neg Q_4)$  (*Propositum* – conceded as following from the denial of 3);
5.  $\neg\neg Q_4$  (*Propositum* – conceded as following from the admission of 2, and the concession of 4);
6.  $Q_4$ .

Then, by strict (necessary) implication, we arrive at

7. If it is that  $P$ , then it is that  $Q_4$ .

**IV.24** In *Quodl.* 5 n. [2-4] 5-11, to propose and to concede consequents following from a *positum* means to think about the contents or necessary and sufficient (definitory) conditions of the *positum* or concept put forward, and in so doing, to substantiate that it is conceivable. Having conceded (C4), Scotus explores further the treatment of infinity as an intensity of entity, and does so through the famous example of “whiteness” (*albedo*)<sup>62</sup>. “Whiteness” is a form of being – an accidental one – and admits of intensity in entity<sup>63</sup>. Surely its degree of intensity in entity is basically ‘finitive’ or basically

to express ontological infinity; since ‘infinity’ bears originally a quantitative sense, when applied to ‘entity’, the quantitative sense is modified to a quantitative category proper to natures, and transcendental concepts such as ‘being’ and ‘perfections’.

<sup>62</sup> Scotus’s metaphysical use of the modal distinction – of “infinitum”/“finitum” – as well as his account of intensive magnitude, were surely inspired by the treatment of modal distinction “in cases of the intensification and remission of forms (...), where some qualitative feature admits continuous variation along a given range: the intensity of color, the amount of heat, the strength of desire, (...)”; see P. King, Scotus on Metaphysics, in: Th. Williams (ed.), *The Cambridge Companion to Duns Scotus*, CUP, Cambridge, 2003, p. 25-26. 32-33.

<sup>63</sup> Again, the notion of “whiteness” together with its “intrinsic mode” or “intensity” form a *conceptus per se*; see Duns Scotus, *Ordinatio* I d. 3 p. 1 q. 1-2 n. 58 p. 40.

determined by it, but even under this fundamental disjunct property or mode, a form of being, compared to other forms, allows quite definite differences in degree/proportion. Scotus states that "whiteness", for example, is exceeded in degree/proportion of (intensity in) entity in these ways: 'triply' (*in triplo*) by another entity such as "strict knowledge" (*scientia*); 'ten times' (*in decuplo*) by the "intellective soul" (*anima intellectiva*); 'a hundredfold times' (*in centuplo*) "by the most perfect angel" (*a supremo angelo*). All these beings are finite and located within a supposed hierarchy of proportion/degree of (intensity in) entity; in all these cases finite beings can be compared in respect of their proportion/degree of (intensity in) entity, and it is always possible to give the "determinate measure" (*proportio determinata*) according to which the 'highest being' (*supremum*) – "the most perfect angel" – exceeds the lowest one; such a determinate measure is always finite<sup>64</sup>.

**IV.25** We have to understand intensities, for infinity is an intensity. Finiteness is also an intensity. Intensities are measurable; they appear in degrees/proportions, and as a kind of quantity they "can be used to 'measure' differences between things"<sup>65</sup>. Therefore, Scotus spends time explaining degrees/proportions of intensities, beginning with the intensity called 'finiteness'. He clarifies that among finite beings, in respect of their comparison, there is no "proportion [degree of intensity in entity] properly speaking" (*proprie proportio*) – that is, there is no proportion in the mathematical sense, e.g., three, ten, or hundred times, etc.. Talk of proportions in this context is thus an analogical way of speaking of a hierarchy among beings concerning their intensity in entity, and here "three, ten, hundred times, etc.", are arbitrary classifications. Taking the angel itself – the supreme being in the hierarchy – Scotus asserts that, being simpler, it is not constituted by some lesser entity to which something has been added in the sense that it gets "more entity" in terms of "more [extensive/additive] quantity of entity". Degrees or proportions of (intensity in) entity, and also the differences in degree or proportion of (intensity in) entity among finite beings,

<sup>64</sup> See Duns Scotus, *Quaestiones quodlibetales* q. 5 n. [4] 9 p. 169: "Verbi gratia, accipiat haec entitas 'albedo'; exceditur ab alia entitate, quae est scientia, in triplo; iterum exceditur ab anima intellectiva in decuplo; iterum a supremo angelo esto quod in centuplo; qualitercumque procedis in entibus, semper esset dare in qua proportione determinata supremum excedit infimum; (...)"

<sup>65</sup> See F. Alluntis and A. B. Wolter, Translation, Introduction, Notes, and Glossary, in: John Duns Scotus, *God and the Creatures – The Quodlibetal Questions*, Princeton University Press, Princeton – London, 1975, p. 110-111 note 4: One sense of quantity "is the strictly proper sense used by the mathematician or physicist (...) which exists between what contemporary philosophers would call *extensive* or *additive* properties like weight, length, time intervals, electric current, etc. The other is analogous to that which holds between *intensive* or *non-additive* differences where these can be arranged serially. We can grade students, for instance, according to intelligence, mastery of a given subject matter, alertness, etc. Numbers can then be assigned to various groups. Thus two in the same percentile may be said to be equal, whereas those in different percentiles are said to be either better or worse than the others. Yet the 'degree of difference' is not an exact mathematical one".

must be understood in terms of the proportion/degree of "power or perfection" (*secundum proportionem virtutis et perfectionis*), in the sense that there is "superiority" (*excessus*) in power or perfection among different species, according to *what they are*<sup>66</sup>. This seems to mean that the existence of different proportions/degrees (of intensity) in entity indicates inferiority or superiority in power or perfection among finite beings – of an intensity in power or perfection that is, in an analogical and mathematically not exact sense, measurable. But the way that a being infinite in entity exists is described through a contrast: a being infinite in entity exceeds a being finite in entity (*in entitate finitum*) beyond every measure that can be assigned (*assignabilem*)<sup>67</sup>. Infinity is an intensity in entity that is not finitely measurable, i.e., whose degree of intensity in entity is 'beyond any determinate and determinable measure'. Having incorporated the language of "degrees" and "proportionalities" among beings into the discourse, it follows now from (IP2) "An infinite in entity exists in actuality", (or assuming what was obtained in (C1), (C2), (C3), and (C4), "A perfect whole of being, with no intrinsic limitation and whose intensity in entity is infinity, exists"), that (C5) "There is a being whose degree of intensity in entity is beyond any determinate or determinable degree of intensity in entity among finite beings".

**IV.26** Let us call "P" the proposition "An infinite in entity exists in actuality" and "Q<sub>5</sub>" the proposition "There is a being whose degree of intensity in entity is beyond any determinate or determinable degree of intensity in entity among finite beings" (Consequent 5 above). Then we have:

1.  $\neg \Diamond P$  (*veritas naturaliter*);
2.  $P$  (*positio impossibilis naturaliter*);
3.  $P \wedge \neg Q_5$  (*Propositum* – denied because it is false; see III.6, III.14, and IV.20-25);
4.  $\neg(P \wedge \neg Q_5)$  (*Propositum* – conceded as following from the denial of 3);
5.  $\neg\neg Q_5$  (*Propositum* – conceded as following from the admission of 2, and the concession of 4);
6.  $Q_5$ .

Then, by strict (necessary) implication, we arrive at

7. If it is that  $P$ , then it is that  $Q_5$ .

**IV.27** As noted above, we confine our analysis to the account of infinite being offered in *Quodlibet* 5. To be sure, there are other sections of

<sup>66</sup> See *Quaestiones quodlibetales* q. 5 n. [4] 9 p. 170: "(...); non quod ibi sit proprie proportio talis quali utuntur mathematici, quia non constat angelus ex aliquo inferiori cum aliquo addito, cum sit simplicior, sed intelligendum est hoc secundum proportionem virtutis et perfectionis, sicut est excessus in speciebus". Here, essences or forms are different things measurable by an *intensive* or *non-additive* quantity: finity and its degrees.

<sup>67</sup> See *Quaestiones quodlibetales* q. 5 n. [4] 9 p. 170: "Hoc modo, per oppositionem, infinitum excedit in entitate finitum ultra omnem proportionem assignabilem".

the text which inspire reflection on the exposition above concerning the distance between the infinite in intensity of entity and the finite in intensity of entity as a *different articulation* of how the actual infinite in entity is a “whole”. In *Quodl.* 5 n. [25-26] 55. 57, Scotus compares the different proportions, and hence, the different intensities in entity in a way that models the comparison of whole and parts, where the whole is not the “quiddity”, and the parts are not “attributes” or “pure perfections” – the “whole” is rather the divine infinite entity, and the “parts” contained in it are, in a special sense, the creatures. Scotus does not bring this account of whole and parts into his construction of a philosophical concept of infinite being. At any rate, the step from (IP2) to (C5) can be viewed as a way of broadening the meaning of *ens infinitum* in terms of a *whole* that contains as *parts* “the totality of entity”, where *parts* are all created beings in a hierarchy such as the one just mentioned (IV.24). Surely the containment of the parts by the whole here is through virtuality<sup>68</sup> and eminence, just as God’s intellect contains and grounds “originally” (*pricipiative*) all ideas that are in themselves formally possible<sup>69</sup> and, by originating them, contains in a supreme way all perfections that they have (including pure perfections). Perhaps the reason for not including explicitly this second consideration of whole/parts in the *positio* is connected with the fact that it is a theological-metaphysical speculation, and as such does not add anything to the quidditative and modal accounts of the divine being infinite in its essence. Scotus affirms in *Quodl.* 5 n. [24-25] 55 that the essence of God, as an “ocean” (*pelagus*), contains all perfections; that is, it contains in the most perfect way every “intrinsic entity” (*entitatem intrinsecam*) that is possible for a single being formally to contain. All things, and not just divine “attributes” and/or “pure perfections”, “emanate” (*emanant*) from the First Entity in a determinate order – (i) first, “intrinsic essential things” (*intrinseca essentialia*), which express no external relations; (ii) second, the notional or personal properties (*notionalia*) in real but not formal identity; (iii) third, “created or extrinsic things” (*creata sive extrinseca*) not in real and/or formal identity. Each thing that emanates from the first essence receives from it the perfection of which it is capable (as long as the perfection is not incompatible with itself), and the effective cause of what is received by creatures is the in-

<sup>68</sup> There are contexts where Scotus explicitly says that the divine essence contains “virtually” the notional or personal properties, such as “paternity”, where two distinct formalities – “deity” and “paternity” – “in one and the same really identical thing” are such that one of them contains virtually the other; see F. ALLUNTIS and A. B. Wolter, *Glossary: virtually*, in: John Duns Scotus, *God and the Creatures – The Quodlibetal Questions*, Princeton University Press, Princeton – London, 1975, p. 538; see also Duns Scotus, *Lectura* I d. 2 q. 1-4 n. 272, ed. Vat., vol. XVI, Civitas Vaticana, 1960, p. 215. But in *Quodlibet* 5 Scotus clearly denies that there are formal criteria for calling the notional property of paternity a “part” of the “whole” or the divine essence.

<sup>69</sup> See Duns Scotus, *Ordinatio* I d. 36 q. un. n. 26-66, ed. Vat., vol. VI, Civitas Vaticana, 1963, p. 281-298.



finitude of the divine intellect<sup>70</sup>; the divine essence thus contains the totality of entity, although not in each case in the same way<sup>71</sup>.

**IV.28** The second model for the relationship of whole and parts finds fullest expression in Scotus's answer to the main argument of *Quodl.* 5 n. 3, which maintains that "the relation of origin [paternity] is formally infinite". Scotus states in *Quodl.* 5 n. [26] 57 that infinity in entity is "totality in entity" (*totalitatem in entitate*), while finitude is "partiality of entity" (*partialitatem entitatis*). Since the finite as such is "less" (*minus*) than the infinite, Scotus invokes Euclid's *Elements* to support the claim that the "smaller number" (*numerus minor*) is a part or parts of the larger one<sup>72</sup>. This account of parts of larger numbers – parts and wholes in mathematical quantity – is analogous to the relation between finite beings and infinite being. Accordingly, no created being is a real or formal part of the essence of God, since God is most simple; nonetheless, every finite being or being that is really different from God, *since it is less than* the infinite entity, can be called a "part": it can be so called not in a determinate proportion, since it is exceeded *ad infinitum* by the divine essence, but in the sense that it is a being "through participation" (*per participationem*). After all, through participation a created being captures a part of the entity that in God is "totally and perfectly" (*totaliter et perfecte*)<sup>73</sup>. Indeed every finite being, because it is "less than" the infinite being, is a "part"; "to be less than something" thus belongs to the meaning of "part". Since it is incompatible with divinity to be part or be really exceeded by anything, finitude is incompatible with it. In this sense, "paternity" (*paternitas*), as a notional property, is no part of the divine essence, because it is incompatible with it to be really exceeded

<sup>70</sup> See Duns Scotus, *Quaestiones quodlibetales* q. 5 n. [24-25] 55 p. 196: "Et cum additur quod infinitas intellectus est propter identitatem eius ad essentiam, [25] respondo: primum omnino in divinis, ut tactum est in prima quaestione, est essentia ut essentia, (...)"

<sup>71</sup> See Duns Scotus, *Quaestiones quodlibetales* q. 5 n. [25] 55 p. 196-197: "Videtur ergo quod essentia habet infinitatem et formaliter et propriam et primam quia a se; quia respectu omnium dicitur pelagus, quia omnem perfectissime continet entitatem intrinsecam, ut possibile est eam contineri in uno formaliter. Ab hac autem prima, sicut licet loqui, emanant omnia ordinate; primo quidem intrinseca essentialia, quae non dicunt respectum ad extra; secundo, notionalia; tertio et ultimo, creata sive extrinseca; et quodlibet emanans recipit illud perfectionis ab ea cuius est capax, si sibi non repugnat; et illius recepti causa quasi effectiva et primaria est infinitas essentiae, (...)"

<sup>72</sup> See Duns Scotus, *Quaestiones quodlibetales* q. 5 n. [26] 57 p. 198-199: "Sicut declaratum est in primo articulo, infinitas in entitate dicit totalitatem in entitate, et per oppositum suo modo finitas dicit partialitatem entitatis; omne enim finitum ut tale minus est infinito ut tali; quia, secundum Euclidem, VII conclusione 4: 'Omnis numerus minor maioris numeri pars est, vel partes'".

<sup>73</sup> See Duns Scotus, *Quaestiones quodlibetales* q. 5 n. [26] 57 p. 199: "Nullum creatum est pars Dei, cum Deus sit simplicissimus, sed omne finitum, cum sit minus illa entitate infinita, conformiter potest dici pars, licet non sit secundum aliquam proportionem determinatam, quia exceditur in infinitum; et hoc modo omne aliud ens ab ente infinito dicitur ens per participationem, quia capit partem illius entitatis, quae est ibi totaliter et perfecte".

by divinity; it cannot be itself “part” or have the “reason of a part” (*rationem partis*). On account of not having either the reason of a part or the reason of a whole, the relation of origin or “paternity” cannot be finite or infinite, since all finite things have the “reason of a part”, and the infinite essence has the “reason of a whole” (*rationem totius*): “from the plenitude of its virtual quantity” (*ex plenitudine quantitatis virtualis*), namely, infinity in intensity of entity, the infinite (the whole) measures every other being “as greater through the approximation, and less through the distance to itself”<sup>74</sup>.

## V. Mode of Being

**V.29** It is only after this “modal description” of a being infinite in entity, according to which infinity in entity is a proportion of intensity described “through [undeterminable] excess” or “[undeterminable] superiority” (*per excessum*) in respect to what is measured or measurable in finite beings – namely their degree of intensity in entity – that Scotus chooses *other words* for ontological infinity and affirms, not that it is “proportion” in terms of “undeterminable excess or superiority of intensity in entity”, but more simply that it is “infinity of intensity” or “intensive infinity” (*infinitas intensiva*). To relate “intensive infinity” or “infinity of intensity” to a “being”, calling it “infinite being”, just as in *Ord.* I d. 8 p. 1 q. 3 n. 108-109<sup>75</sup>, (a) is not to speak of ‘an attribute’ (*passio*) that comes to it extrinsically (*passio extrinseca adveniens*), (b) nor to speak of something of a “being” in the manner of “convertible” transcendental properties such as “one”, “true”, and “good”, which are formally *a parte rei* non-identical with “being”, and (c) as we surely could infer (Scotus does not himself infer it here, but later does, calling it an ‘attributable property’ (*proprietates attributalis*)<sup>76</sup> nor is it to speak of a ‘pure perfection’ (*perfectio simpliciter*) of being such as “wisdom” (*sapientia*) and “goodness” (*bonitas*). To relate “intensive infinity” or “infinity of intensity” to a being, calling it infinite being, is to say that it is an “intrinsic mode” of that entity or being (*infinitas intensiva dicit modum*

<sup>74</sup> See Duns Scotus, *Quaestiones quodlibetales* q. 5 n. [26] 57 p. 199-200: “Hoc ergo volo habere, quod omne finitum, cum sit minus infinito, est pars; cui ergo repugnat esse pars, vel excedi realiter ab aliquo, ei repugnat esse finitum; nunc autem paternitati huius repugnat esse pars divinitatis illo modo, vel excedi a divinitate, quia propter infinitatem divinitatis paternitas, cum sit compossibilis sibi in eodem supposito, est simpliciter idem sibi; et, per consequens, realiter excedi non potest, nec esse pars, nec rationem partis habere potest. Non ergo est finita nec infinita, sicut prius probatum est, quia sicut finitum habet rationem partis modo praedicto, ita infinitum habet rationem totius hoc modo, scilicet ex plenitudine quantitatis virtualis suae mensurans omne aliud ut maius per accessum ad ipsum, et minus per recessum; sed nec paternitas rationem totalitatis, (...)”.

<sup>75</sup> See Duns Scotus, *Ordinatio* I d. 8 p. 1 q. 3 n. 108-109 p. 202-203.

<sup>76</sup> In *Ordinatio* I d. 8 p. 1 q. 4 n. 162 p. 233, Scotus speaks of a ‘pluralitas perfectionum attributalium’.

*intrinsicum illius entitatis*) as it is conceivable on the basis of its nature<sup>77</sup>. Scotus here explains ‘modus intrinsicus’ in terms of ‘intensio’ or ‘intentio’<sup>78</sup>: the intensity in entity of a being is its intrinsic mode<sup>79</sup>. Accordingly, it follows from (IP2) “An infinite in entity exists in actuality”, or, assuming what was obtained before in (C1), (C2), (C3), (C4), and (C5), i.e., “A perfect whole of being, with no intrinsic limitation and whose intensity in entity is infinite or beyond any determinate or determinable degree of intensity in entity among finite beings, exists”), that (C6) “There is a being whose intrinsic mode in entity is infinity”.

**V.30** Let us call “P” the proposition “An infinite in entity exists in actuality” and “Q<sub>6</sub>” the proposition “There is a being whose intrinsic mode in entity is infinity” (Consequent 6 above). Then we have:

1.  $\neg\Diamond P$  (*veritas naturaliter*);
2.  $P$  (*positio impossibilis naturaliter*);
3.  $P \wedge \neg Q_6$  (*Propositum* – denied because it is false; see III.6, III.14, IV.17-26, and V.29);
4.  $\neg(P \wedge \neg Q_6)$  (*Propositum* – conceded as following from the denial of 3);
5.  $\neg\neg Q_6$  (*Propositum* – conceded as following from the admission of 2, and the concession of 4);
6.  $Q_6$ .

Then, by strict (necessary) implication, we arrive at

7. If it is that  $P$ , then it is that  $Q_6$ .

**V.31** Having passed from intensity in entity to mode of being or of entity, Scotus now explores the idea of “intrinsic mode” and its difference from “being”/“entity”; this exploration points to a distinction between “mode” and “attribute”/“perfection” concerning the way that the latter attaches to “being” or “entity”. In what follows, it is plain that Scotus makes use of insights from *Ordinatio* I d. 8 p. 1 q. 3 n. 136-150. Accordingly, an intensity or an intrinsic mode is so intrinsic to a being that, if we abstract from all properties or quasi-properties of that being, we have not yet excluded, say, infinity from it, but it remains integrally included in that single entity itself. “Intrinsic” appears here as “inseparable”, for something “separable” would

<sup>77</sup> See *Quaestiones quodlibetales* q. 5 n. [4] 10 p. 170: “Ex hoc sequitur quod infinitas intensiva non sic se habet ad ens, quod dicitur infinitum, tanquam quaedam passio extrinseca adveniens illi enti; nec etiam eo modo quo ‘verum’ et ‘bonum’ intelliguntur passiones vel proprietatis entis, imo infinitas intensiva dicit modum intrinsicum illius entitatis, (...)”.

<sup>78</sup> Scotus twice uses the word ‘intensio’ in *Quaestiones quodlibetales* q. 5 n. [23] 52 p. 195, where it is contrasted to ‘extensio’. We also find the synonym ‘intentio’ in other works; see, for example, Duns Scotus, *Opera philosophica IV – Quaestiones super libros metaphysicorum Aristotelis Libri VI-IX*, lib. VII q. 15 n. 20, Ed. R. Andrews et alii, Franciscan Institute Publications, St. Bonaventure (N. Y.), 1997, p. 301.

<sup>79</sup> This is explained in more detail in: R. H. Pich, *Infinity and Intrinsic Mode*, op. cit., p. 210-215.

indeed be “extrinsic”<sup>80</sup>. An intrinsic mode is not and cannot be formally distinguished from the being/nature of which it is a mode (as it would be distinguished in a *distinctio formalis* between two “realities”<sup>81</sup>). Actually, as pointed out above, to conceive the entity itself under this mode is simply to apprehend it fully<sup>82</sup>. Scotus affirms that if we consider that entity “most precisely” (*praecisissime accepta*) – in itself and without any property – it will still be true to say that it has a “proper” or “intrinsic” “magnitude” (*magnitudinem propriam*) “of power” (*virtutis*)<sup>83</sup>. Scotus changes the descriptive expressions, putting his discourse in more categorical terms; he makes sure that an infinite “being”/“entity” according to its nature has an “intrinsic magnitude” – comparable to an “intrinsic mode” and to a proper “intensity in entity” – that is “infinite”, since its nature is incompatible with an “intrinsic magnitude” that is “finite”. The language of “intensity”, “mode” or “magnitude” of entity is *intrinsic* or *proper*, since it belongs entirely to the way the “entity” or “nature” is; the language of *intensity*, *intrinsic mode* or *intrinsic magnitude* admits fundamentally the degrees of *infinity* and *finity*<sup>84</sup>.

**V.32** If Scotus says that a being or an entity “taken most precisely” has an “intrinsic magnitude” or an “intrinsic mode”, he can now express what (only) “infinite” (*infinitum*) is, “considered most precisely” (*praecisissime acceptum*). He wants to describe in a more satisfactory way what an intrinsic mode or an intrinsic magnitude is, or at least what *this* intrinsic mode or intrinsic magnitude called “infinite” or “infinity” is. “To describe” is a correct expression here, for human beings have no full concept or understanding of a nature that shows what it is under the mode of infinity. We only have means for constructing a consistent concept of “infinite being” or “infinite entity”, once we concede the *positum* (IP2) “An infinite in entity exists in actuality”. Scotus wants to distinguish precisely *infinitum* from attributable properties – i.e., pure perfections, as indicated in V.29 – but this time *he does not simply classify* “infinite” as a mode of being/entity; he rather reconsiders it as an intensity in entity that has a degree/proportion and hence can be compared (*comparari*), according to an essential order of

<sup>80</sup> See G. Sondag, Jean de Damas et Jean Duns Scot sur l’infinité de l’essence divine, op. cit., p. 312.

<sup>81</sup> See Duns Scotus, *Ordinatio* I d. 8 p. 1 q. 3 n. 139 p. 222-223.

<sup>82</sup> See above under IV.20 (note 53); see Duns Scotus, *Ordinatio* I d. 8 p. 1 q. 3 n. 138 p. 222. See also P. King, Scotus on Metaphysics, op. cit., p. 25-26.

<sup>83</sup> Surely the expressions ‘magnitudo propria’, ‘magnitudo intrinseca’, ‘modus proprius’, and ‘modus intrinsecus’ may be viewed as synonyms. See again R. H. Pich, *Infinity and Intrinsic Mode*, op. cit., p. 210-214.

<sup>84</sup> See Duns Scotus, *Quaestiones quodlibetales* q. 5 n. 4 [10] p. 170: “(…), cuius est sic intrinsecum, quod circumscribendo quodlibet quod est proprietas vel quasi proprietas eius, adhuc infinitas eius non excluditur, sed includitur in ipsa entitate, quae est unica. Vnde de ipsa entitate praecisissime accepta, absque scilicet quacumque proprietate, verum est dicere quod aliquam magnitudinem propriam virtutis habet sibi intrinsecam, et non magnitudinem finitam, quia ipsa repugnat sibi; ergo infinitam”.

beings mentioned in IV.24, with things that it simply excels. *Infinitem* as intensity allows the degree/proportion of purely excelling any thing. 'Pure excelling' (my expression) is a relative expression that indicates that something "excels", "is superior" to something else, (i) not according to a degree/proportion in any defined scale – else *infinitem* would be a finite degree or proportion in entity – but rather, we can infer, in a manner that is (ii) beyond any determinate or determinable degree<sup>85</sup>. Such a final expression for "infinite" being or "infinity" in entity *does not bring any new information*, but allows that we clearly equate "intensity" and "intrinsic mode of being", for it follows from (IP2) "An infinite in entity exists in actuality" (when we assume what was obtained before in (C1)-(C6)) that (C7) "There is a being whose degree of intensity in entity or whose intrinsic mode of being is beyond any determinate or determinable degree of intensity in entity or intrinsic mode of being among finite beings".

**V.33** Let us call "P" the proposition "An infinite in entity exists in actuality" and "Q<sub>7</sub>" the proposition "There is a being whose degree of intensity in entity or whose intrinsic mode of being is beyond any determinate or determinable degree of intensity in entity or intrinsic mode of being among finite beings" (Consequent 7 above). Then we have:

1.  $\neg\Diamond P$  (*veritas naturaliter*);
2.  $P$  (*positio impossibilis naturaliter*);
3.  $P \wedge \neg Q_7$  (*Propositum* – denied because it is false; see III.6, III.14, IV.17-26, and V.29-32);
4.  $\neg(P \wedge \neg Q_7)$  (*Propositum* – conceded as following from the denial of 3);
5.  $\neg\neg Q_7$  (*Propositum* – conceded as following from the admission of 2, and the concession of 4);
6.  $Q_7$ .

Then, by strict (necessary) implication, we arrive at

7. If it is that  $P$ , then it is that  $Q_7$ .

## VI. Final Results of the *impossibile positum*

**VI.34** Scotus can now say directly what actual infinity is. This is no surprise; after all, he employed *positio impossibilis* for the purpose of forming a concept of *actual infinity*, which he maintains is the concept of *ontological infinity* alone. If there was any doubt about the continuity of the *positio* when Scotus began the quidditative and modal descriptions (IV.19ff.), there is now a clear indication that the intended infinity receives its final

<sup>85</sup> See *Quaestiones quodlibetales* q. 5 n. [4] 10 p. 170: "Ipsum etiam infinitum, praecisissime acceptum non sub aliqua ratione proprietatis attributalis ut bonitatis vel sapientiae, potest comparari secundum ordinem essentialem ad aliqua quae excedit, et non secundum aliquam proportionem determinatam, quia tunc esset finitum; (...)".

constitution only after the account of actual infinity obtained on the basis of Aristotle's potential infinite (see I.1-III.18) and the elucidation of the idea of mode of being/entity (see IV.22ff. and V.29ff.). Scotus affirms that "infinity itself" (*ipsa infinitas*) can be defined as the "intrinsic mode" of any infinite being in intensity or of any being which, because of its nature, is infinite in intensity. To be sure, some difficulties attend this passage in the *Quodlibet* 5; it suggests, for example, that we can think of infinity in terms of intrinsic mode and intensity so long as we keep in mind that infinity is a fundamental measure (analogous to quantity) in entity. However, at the same time, it maintains that «actual infinity» and «ontological infinity», including a «modal» or even «quantitative»<sup>86</sup> description, must be offered in terms of an *entity* or a *nature* that is infinite; its mode is attached to it in a way that admits no formal distinction between nature and mode<sup>87</sup>. There is no way of constructing an acceptable concept of actual and ontological infinite save by combining quidditative and modal descriptions. To put the works of Scotus in perspective, we may observe that what is really new in *Quodlibet* 5 is, first, a quidditative description that has been built<sup>88</sup> carefully upon Aristotle's mathematical account of potential infinity and, second, the clarification of a language of ontological modes that allows for the transfinite. Scotus masterfully summarizes these ideas by stating at the end of *Quodl.* 5 n. [4] 10 that «infinity» denotes intrinsically just that – i.e., an *essence* – (I) which lacks nothing (that is, nothing in entity, where we can read again the notions of strict whole and strict perfect) and (II) which, as *entity* and as *attributes* that have no intrinsic limitation, admits an intensity (i.e., a mode or a magnitude of entity) that exceeds everything finite beyond every determinable proportion or degree (where we again read the terms of the modal description)<sup>89</sup>. It thus follows from (IP2) «An infinite in entity exists in actuality» (assuming what was obtained before in (C1)-(C7)), that (C8) «Ontological infinity cannot help but be (I) strict whole and strict perfect of entity and attributes of entity with no intrinsic limitation, and (II) an intrinsic mode, a magnitude of power or even an intensity in infinity, whose degree is purely excess».

**VI.35** Finally, let us call «P» the proposition «An infinite in entity exists in actuality» and «Q<sub>8</sub>» the proposition «Ontological infinity cannot help but be (I) strict whole and strict perfect of entity and attributes of entity with no intrinsic limitation, and (II) an intrinsic mode, a magnitude of

<sup>86</sup> I deal with ontological «quantitative degrees» or with *magnitudo/quantitas virtutis* and *modus intrinsecus* for essences or natures in another study; see R. H. Pich, *Infinity and Intrinsic Mode*, op. cit., p. 159-214. See also *Ordinatio* I d. 19 q. 1 n. 8, ed. Vat., vol. V, Civitas Vaticana, 1959, p. 267.

<sup>87</sup> See the references in notes 75, 76, 81, 82.

<sup>88</sup> Differently than in Duns Scotus, *Ordinatio* I d. 2 p. 1 q. 1-2 n. 132 p. 206-207.

<sup>89</sup> See *Quaestiones quodlibetales* q. 5 n. [4] 10 p. 170-171: «(...); intrinsecus ergo modus cuiuslibet infiniti intensive est ipsa infinitas, quae intrinsece dicit ipsam esse cui nihil deest et quod excedit omne finitum ultra omnem proportionem determinabilem».

power or even an intensity in infinity, whose degree is purely excess» (Consequent 8 above). Then we have:

1.  $\neg\Diamond P$  (*veritas naturaliter*);
2.  $P$  (*positio impossibilis naturaliter*);
3.  $P \wedge \neg Q_8$  (*Propositum* – denied because it is false; see III.6, III.14, IV.17-26, and V.29-34);
4.  $\neg(P \wedge \neg Q_8)$  (*Propositum* – conceded as following from the denial of 3);
5.  $\neg\neg Q_8$  (*Propositum* – conceded as following from the admission of 2, and the concession of 4);
6.  $Q_8$ .

Then, by strict (necessary) implication, we arrive at  
7. If it is that  $P$ , then it is that  $Q_8$ .

**VI.36** It is worth noting that Scotus seems to treat the definition in VI.35, which he corroborates with an authority from John Damascene, as a corollary of ontological infinity; infinite being – or better an “essence” (*essentia*) that is infinite – is compared to an “infinite”/“endless” (*infinitum*) and “limitless”/“unlimited” (*interminatum*) “ocean of substance” (*pelagus substantiae*). As already pointed out, *infinitum* and *interminatum*<sup>90</sup> can be taken as synonyms<sup>91</sup>. At the same time, it seems warranted to interpret their uses – namely “limitless” and “limitlessness”, as well as “infinite” and “infinity” of an essence – as expressing precisely the true idea of *infinitas in actu*, an idea to be understood in terms of (I) strict whole and strict perfect in entity and (II) intensity or intrinsic mode under a degree of undeterminable measure. In this case, divine substance itself, as long as it has the structure of what is absolutely first in the divine reality, and is called an “ocean” by John Damascene, is “infinite and limitless” (*infinita et interminata*); it is exactly in this sense that an infinite being is the divine being or substance, and the concept of infinite being can work as the concept of the divine being or substance<sup>92</sup>. Nevertheless, it should be emphasized that Duns Scotus makes use of the expressions inspired by John Damascene in order to confirm his thesis that in considering the substance understood (perfectly) as “infinite”, he does not include any attributable properties such as truth, goodness, life, etc., but rather substance as such. For this reason, “infinity itself” (*ipsa infinitas*) or the notion of “infinity” as such and by it-

<sup>90</sup> For its part, ‘interminatum’ in Latin means the same as ‘illimitatum’.

<sup>91</sup> See above under III.9.

<sup>92</sup> And it is possibly because of the intended combination of quidditative and modal aspects that Scotus hesitates between the expressions ‘pelagus infinitum substantiae’ (indeed found in John Damascene and here in *Quodlibet* 5) and ‘pelagus infinitae substantiae’ (as Scotus puts it in other contexts). See F. J. S. Catania, op. cit., pp. 47-48. See also Duns Scotus, *Ordinatio* I d. 8 p. 1 q. 4 n. 198 p. 264: “Ista opinio confirmatur auctoritate Damasceni cap. 4 praeallegato, et cap. 9, ubi ipse vult quod inter omnia nomina de Deo dicta, propriissimum est *Qui est*, quia esse dicit ‘quoddam pelagus infinitae substantiae’; (...)”.

self expresses an intrinsic mode of an essence more than an attribute of an essence; it expresses just *how* the essence is in what is intrinsically unlimited within it<sup>93</sup>.

## VII. Conclusion: *positio impossibilis* and Concept Formation

VII.37 Departing from this case study on the use of *positio impossibilis*, it is time to conclude how *positio* itself and the *ars obligatoria* are to be understood for Scotus. Since the *positio* was employed both for the analysis and the construction of the concept of infinite being as opposed to a concept of infinite quantity, and in particular on the basis of reasoning *secundum imaginationem*, I see no reason to compare what Scotus does either to an account of the *ars* in terms of the “counterfactual hypothesis” (P. V. Spade)<sup>94</sup> or in terms of the “consistency game hypothesis” (C. Dutilh Novaes)<sup>95</sup> – interpretations that could find support in studies focused on logical treatises *De obligationibus* as the subjects of investigation<sup>96</sup>. While I think that Ch. Martin’s criticism of Spade’s “would-conditionals” in favor of a “might-conditionals” account (where the emphasis rests on the evaluation of the co-tenability between propositions or the sense how anything compossible with the *positum* can be proved, or “might” be the case, if the *positum* were true) is sound and connects the *ars* to the construction of possible worlds by means similar to contemporary systems in modal logics<sup>97</sup>, I am nevertheless

<sup>93</sup> See *Quaestiones quodlibetales* q. 5 n. [4] 11 p. 171: “Et istud corollarium confirmatur per Damascenum cap. 7, ubi ipse vult quod essentia dicit pelagus substantiae infinitum et interminatum. Substantia ergo, secundum quod habet rationem omnino primi in divinis et vocatur ab eo pelagus, sic ipsa est infinita et interminata; sic autem non includitur in ea nec veritas, nec bonitas, nec aliqua proprietates attributalis; ergo ipsa infinitas est magis modus intrinsecus essentiae, quam aliquod attributum”. See also G. Sondag, *Duns Scot. La métaphysique de la singularité*, Vrin, Paris, 2005, p. 115ff.

<sup>94</sup> See, for instance, P. V. Spade, Three Theories of Obligations: Burley, Kilvington and Swyneshed on Counterfactual Reasoning, in: *History and Philosophy of Logic*, 3 (1982), p. 1-32.

<sup>95</sup> Where *obligationes* can sometimes be viewed (as in Roger Swyneshed’s *Obligationes*) as a strategy for recognizing inferential relations between propositions – particularly between the *positum* and the propositions proposed each time (see C. Dutilh Novaes, Roger Swyneshed’s *Obligationes*: a Logical Game of Inference Recognition?, in: *Synthese*, 151:1 (2006), p. 125-153), but more fundamentally (as in Walter Burley’s *Obligationes*) as “logical games of consistency” (see C. Dutilh Novaes, *Medieval Obligationes* as Logical Games of Consistency Maintenance, op. cit., p. 371-395).

<sup>96</sup> For surveys of the literature and the various interpretations of *obligationes* see C. Dutilh Novaes, *Medieval Obligationes* as Logical Games of Consistency Maintenance, op. cit., p. 373-376; Ch. Pütz, *Die Obligationenlehre in der scholastischen Logik, Untersuchungen zum Einfluß der stoischen Logik auf die Lehre von den Verpflichtungen (De obligationibus)*, Universität Düsseldorf, Düsseldorf, 1997, p. 72-106.

<sup>97</sup> See Ch. J. Martin, Obligations and Liars, in: S. Read (ed.), *Sophisms in Medieval Logic and Grammar*, Kluwer, Dordrecht, 1992, p. 357-381.



inclined to agree, in respect of Scotus's *positio*, with M. Yrjönsuuri's explanation of *obligationes* as "thought experiments"<sup>98</sup>.

**VII.38** Understood as thought experiments<sup>99</sup> – as an assumption of a (counter)possible situation in order to learn from it "something applicable generally"<sup>100</sup> – *ars obligatoria* and particularly *positio impossibilis* show no strong resemblance to classical counterfactual reasoning (i.e., D. Lewis's interpretation of the hypothetical conditional)<sup>101</sup>. That being said, they must of course manifest consistency and agreement with logical rules. Our reconstruction of Scotus's *positio*<sup>102</sup> makes this plain; moreover, we see in it a process of reasoning that starts from a situation contrary to natural possibilities, and yet without evidence of any need to remain as close as possible to the actual world. In this sense, counterfactual reasoning would not appear to be fruitful for pure conceptual investigations, such as the study of the concept of *ens infinitum*, since in such investigations it is both permissible and necessary to depart as far as possible from the actual world<sup>103</sup>. Even though I cannot defend that this is generally true for a theory of obligations as a logical-technical exercise, I do think that it applies to Scotus in *Quodlibet* 5.

**VII.39** At a minimum, the thought experiment at issue – the conception of an actual infinite being grounded on a *positio secundum imaginationem* – can be connected to aspects ("anachronistic" as they may be) of possible world semantics<sup>104</sup>, which is itself related to the analysis of modal concepts but also shows that logical possibilities can be used to conceive something and hence to construct concepts. At the root of Scotus's achieve-

<sup>98</sup> See M. Yrjönsuuri, *Obligations as Thought Experiments*, in: I. Angelelli and M. Cerezo (eds.), *Studies on the History of Logic*, Walter de Gruyter, Berlin, 1996, p. 93. I see no difficulty with maintaining that such an account coheres with the underlying notion of consistency maintenance for the exercises of *positio*.

<sup>99</sup> Because it emphasizes conceivability and logical possibilities (see also below in the main text, M. Yrjönsuuri's account of *obligationes*, particularly of *positio impossibilis*), it can be seen as a "more epistemological version of Martin's idea that an obligational disputation mirrors the construction of a state of affairs (or a possible world)"; see C. Dutilh Novaes, *Medieval Obligations as Logical Games of Consistency Maintenance*, op. cit., p. 375.

<sup>100</sup> See M. Yrjönsuuri, *Obligations as Thought Experiments*, op. cit., p. 85: "Obligations are not of much value in terms of preparation to possible situations, neither in prospect of finding oneself in ethically problematic situations, nor in relation to application of concepts of natural philosophy in practice. I believe that the anonymous author [of *De arte obligatoria*, Merton college ms. 306] rather had in mind the methodology of taking up possible situations in order to learn from them something applicable generally. This methodology is nowadays called thought experiments".

<sup>101</sup> See D. Lewis, *Counterfactuals*, Oxford, Blackwell, 1973.

<sup>102</sup> See paragraphs 15, 20, 23, 26, 30, 33, and 35.

<sup>103</sup> See M. Yrjönsuuri, *Obligations – 14<sup>th</sup> Century Logic of Disputational Duties*, p. 17, 174-176.

<sup>104</sup> In his many studies, M. Yrjönsuuri works out this interpretation, especially in respect of Burley's *Obligations*.

ment is the idea that the *positio* or possible proposition is false according to all we know, on the basis of the philosophy of nature, about reality or the actual world, although it can be true in some other situation (i.e., in some other world); I accept that a proposition is true in a world iff it signifies some state of affairs that obtains in the world. In this sense, Scotus would see (IP1) and (IP2)<sup>105</sup> as false and possible propositions belonging, say, to the actual world *W* of the disputation or the thought experiment, whereas he would see them as true and possible propositions belonging to another world. This is how a *respondens*, or Scotus himself in his own exercise, would think of the *positio*. If the *positum* in the terms of possible world semantics is a proposition about *some world*, not about *the actual one* (also because one cannot say that for all one knows about the world *W*, and even for all that can be naturally known about it, the *positum* is true at *W*), then the *positum* itself must be logically possible and conceivable – not something self-contradictory, but an *opinabile*. And so it can at least *prima facie* be rationally accepted or thought of, in order for the *respondens* to see what follows from the assumption. This is equivalent to saying that the *positio* must be true in some imaginable world or worlds, which do not have to be possible in respect of the actual world. After all, “(...) in conceptual and logical issues actual facts have no privileged status. Conceptually or logically possible states of affairs can serve as evidence just as actual facts”<sup>106</sup>. I think this is exactly what happens when Scotus changes from “It is not possible, in actuality, that there is an infinite in quantity” (*verum naturaliter*) to “Actually, there is an infinite in quantity” (*impossibile naturaliter positum*) in II.5 (the first *positum*), and especially when he changes from “It is not possible, in actuality, that there is an infinite being” (*verum naturaliter*) to “Actually, there is an infinite in entity” (*impossibile naturaliter positum*) in III.10 (the second *positum*); the assumption of a logically possible world constitutes an immediate admission of conceivability right at the start, although it is still necessary to follow from there (see further below) the stipulation of some consequents as “definitory” aspects or necessary *and* sufficient conditions of a given complex conceptual assumption.

**VII.40** Scholars investigating primarily the nature of *ars obligatoria*, such as like M. Yrjönsuuri, recognize the potential of this technique of inference, particularly in the form of *positio impossibilis*, to generate “something more interesting for philosophy”<sup>107</sup>. If obligations can be used to speculate about a situation that is possible but not actual – or in Scotus’s case about a situation that is at least naturally impossible to know as actual (which amounts to the same) – then it should come as no surprise that

<sup>105</sup> See II.5 and III.10 above.

<sup>106</sup> See M. Yrjönsuuri, *Obligations as Thought Experiments*, op. cit., p. 85.

<sup>107</sup> See M. Yrjönsuuri, *The Role of Casus in Some Fourteenth Century Treatises on Sophismata and Obligations*, in: K. Jacobi (Hrsg.), *Argumentationstheorie. Scholastische Forschungen zu den logischen und semantischen Regeln korrekten Folgerns*, E. J. Brill, Leiden, 1993, p. 320.

it was used in this way by jurists and natural philosophers in 14<sup>th</sup>-century; for his part, Scotus, we may infer, uses it for metaphysical purposes. And while we must admit that it is not always clear what should be understood by the “*secundum imaginationem*-method” of 14<sup>th</sup>-century natural philosophers, Scotus’s usage of it confirms the hypothesis that the admission of a *casus/positio* is a central part of that method of analysis. As other philosophers made use of *casus/positio* to analyze not exactly “physical truths” but “linguistic truths about physical concepts”<sup>108</sup> – that is, to analyze what are not exactly “physical modalities” so much as the “conceptual modalities or the modality of possibility at different levels”<sup>109</sup> – I have myself no doubt that Scotus in *Quodlibet* 5 makes use of a *positum* not primarily to illumine what is a metaphysical truth<sup>110</sup> but rather to illumine a linguistic or conceptual truth about a metaphysical concept: *ens infinitum*.

**VII.41** In this “metalinguistic” sense of *positio impossibilis*, the knowledge that Scotus envisages is not about an existing thing but about a complex concept signifying a thing. In the case study (see I-VI) this is most likely the practical purpose of the logic of obligations. In some of his studies, M. Yrjönsuuri refers to an anonymous author [of *De arte obligatoria*, Merton college ms. 306] who suggests that obligations were also used in the linguistic analysis found more typically in the *sophismata*-literature, where for example physical concepts were clarified by obligational reasoning connected to some *casus*. Scotus’s thought experiment is an attempt to clarify the conceptual assumption of an actual infinity in the *positum*, investigating then what that concept must possibly signify if it is meaningful at all – what must be true in all situations corresponding to the notion proposed in the *positio* – *the actual infinite being*. As I see it, Scotus’s *positio* is restricted to logical-semantic necessity; it concerns no natural necessities of the sort that are often ascribed to the role of *casus* in the *sophismata*. This may confirm the idea that the role of *casus* in *sophismata* literature helps to understand the development of the role of *positum* in *obligationes*<sup>111</sup>. The original ‘*rei veritas*’ assumption of *casus* – and the discussion of what must be true in situations fulfilling what the *casus* describes – finds an echo in the *positio impossibilis naturaliter* as a mere proposition, and not the things signified by it, which find themselves outside of the realm of known possibilities in nature.

**VII.42** The study of Scotus’s *positio* in *Quodlibet* 5 conforms to the rule that whatever is entailed by the *positum* must be granted and that what-

<sup>108</sup> Id. *ibid.*

<sup>109</sup> See M. Yrjönsuuri, *The Compossibility of Impossibilities and Ars Obligatoria*, in: *History and Philosophy of Logic*, 19 (1998), p. 243ff.

<sup>110</sup> That there exists in actuality an infinite being.

<sup>111</sup> See M. Yrjönsuuri, *The Role of Casus in Some Fourteenth-Century Treatises on Sophismata ad Obligations*, *op. cit.*, p. 320-321; *Id.*, *Obligations as Thought Experiments*, *op. cit.*, p. 89-91.

ever is incompatible with the *positum* must be denied; the *respondens* must deny any proposition that is false in the world or worlds specified by the *positum* and what is *pertinens* to it. Accordingly, I think that the propositions formulated in the study as consequents (C1)-(C8) are all “relevant propositions”<sup>112</sup>, and, as emerges from the text, they were found to be connected deductively to the worlds in which a *positum* like (IP2) is true only after careful (even “dialectical”) examination. In order to formulate relevant *proposita*, and for the purpose of showing what it is to fulfill completely a *positum* – and in so doing to construct exactly the complex concept put forward in it – Scotus works very much with conceptual intuitions (such as ‘whole’, ‘perfect’, ‘complete’, ‘entity’, ‘intensity’, ‘intrinsic mode’, ‘illimitation’, etc.). This amounts to a kind of revision and at the same time expansion of a given *positum*. In this sense, in the finding of relevant *proposita* and consequents, further concessions are involved and dialectically worked out. In contradistinction to what happens in counterfactual reasoning, such relevant propositions do not describe but rather construct a possible situation – they construct a concept or show what its very conceivability *is*. This seems to justify and indeed to explain the use of ‘ubi est possibilis’ – that is, the recognition of the logical possibility of contents<sup>113</sup> in III.16 (*Quodl.* 5 n. [3] 8). After that *locus*, it is clear that the clause ‘ubi est possibilis’ rules over the entire (IP2), showing that it is actually possible to find out, after a careful search for the right terms and the right understanding of them, an acceptance of consequents that reveal what belongs essentially to the antecedent. And this should justify the account of *Impossibile Positum 2* to Consequents C1-C8 as *strict implications*, revealing the *logical-semantic necessity* between antecedent and consequent.

**VII.43** What then is the difference between the *Impossibile Positum 1* and the *Impossibile Positum 2* – the first expressing some “subjunctive conditionalization”, and the second a *strict implication*? Both may be seen as “thought experiments”, and Scotus employs both of them by way of discharging the duty of granting what was posited. But surely in the first “game”, he cannot go farther than the two consequents (C1) and (C2); the technique has to end up there (see III.13). To be sure, the continuation of the exercise would demand the denial of the first *positum*; in other words, the second *positum* and its development make evident at the end – or retrospectively – that the first *positum*, although it can be mentally “enter-

<sup>112</sup> It would be correct to affirm that “irrelevant sentences”, which are evaluated according to semantical considerations based on the actual situation, play no role in what Scotus does in *Quodl.* 5 n. [2-4] 5-11. Because the semantical background of the *positum* and of the irrelevant sentences would differ, it would be difficult, from the perspective of contemporary discussions of such matters, to accept a “conditional” account of *obligationes*; after all, “no conditional makes sense with different semantics for the antecedent and for the consequent”; see M. Yrjönsuuri, *Obligations as Thought Experiments*, op. cit., p. 93-94.

<sup>113</sup> I.e., of actual infinity in entity.

tained" in a vague way and without apparent contradiction, nevertheless leads to contradiction and is itself inconsistent. As such, it is properly inconceivable. An actual infinite in quantity, insofar as it is not a strict whole and a strict perfect, must be classified retrospectively as a *contradictio in adiecto*, even if this contradiction is not apparent at a first glance.

**VII.44** The difference between (IP1) and (IP2) then would be not a distinct understanding of *positio* as a thought experiment and metalinguistic analysis (not even of the conditional of strict implication revealed by the examination of (IP2)). The idea is that, being by revision incapable of conceivability, the first *positum* can at most be connected with the given *proposita* in a way similar to subjunctive reasoning. It is thus interpreted like a *casus* or a simple counterfactual assumption, and through (C1) and (C2), there is an attempt to discover what would be true in the situation of (IP1). In a sense, (C1) and (C2) would express necessary conditions for the satisfaction of parts of the elements of the *positum*, but by no means all necessary and sufficient conditions of its complex structure. As an intended *positum*, it is unfit to be investigated in an "imaginable world" since there cannot be an imaginable world for it in its entirety. Keeping the obligation in respect to the first *positum*, Scotus only finds a necessary condition: "There is an actual infinity in quantity, only if there is a whole and a perfect in the respective category". This is semantically acceptable. But the revision of the "necessary condition" achieved through the experiment demands the beginning of a new game, or the discovery of a *positum* in which, in the indicative use of *posse*, a possible world is found indeed; in it, the complex content of the antecedent does work as a function for some value.

**VII.45** In conclusion, I offer two remarks on the idea of concept formation. Concept formation presupposes concepts, some of which may be simple, but most of which are surely non-simple. Scotus looks for a concept that is not obtained by abstraction but offers the basic meaning or structure for thinking of some thing, *deitas*, concerning which we, at least before a successful *proof* of its existence, do not know whether it *exists* or even in some minimal way *what* it is, and for which we have no more than a *quid nominis*. "Infinite being" as a concept is a conscious and artificial organization of thought which proceeds from intuitions characterized by generality and vagueness (such as 'being' and 'infinite', respectively). As a result of intellectual effort, it should be understood not as a constitutive rule but as an ampliative and correcting rule of thought. To 'possess' a concept is different than to 'possess' a linguistic use for it of the sort we possess for 'infinite being'. Scotus surely admits conceptual analysis of linguistic uses and of intuitions, and this certainly can lead to revision and expansion, just as when we analyze "truth", "free will", "justice", etc. Concept formation presupposes *some* constitutiveness (of 'infinite' in terms of 'whole' and 'perfect') and also *creates* some constitutiveness (of 'actual infinite' in terms of 'whole' and 'perfect' with no intrinsic limitation). Scotus's concept formation

through *positio impossibilis* seems to follow linguistic use (say, in theology), whose origin is not necessarily revelation, and then proceeds to articulate different general and vague notions such as “entity”, “infinity”, “totality”, “perfection”, “illimitation”, etc. Concept formation means examination and organization of linguistic use and, of course, of some previous concept possession. In this sense, concept formation is a constructive technique of conceivability. What is it “to conceive” in accordance with the technique practiced above? It is to accept in thought what is thinkable – something not explicitly contradictory (the practitioner must only be able to accept it if he himself thinks it is not contradictory) – even when we do not know whether it is or can be true, and even when we do not have true knowledge of it. From there, it is to undertake an examination in order to see what follows, while (i) correcting or expanding what was conceded and (ii) establishing conditions for concessions and, in so doing, defining or forming a conception that we did not previously have.

**VII.46** I do not know how to answer the question whether and in what measure concept formation can become concept possession. I tend to think that the use of “imaginable worlds” in cases of *positio impossibilis* brings Scotus near to modal-semantic analyses; I also tend to think that such concepts are intensional entities. They could be a subject for formal accounts of the semantics of a natural language in terms of intensions, regarded as functions that map a possible world to the extension of the concept in that world (comparable to what was developed by philosophers such as D. K. Lewis, R. Montague, and R. Stalnaker). But that is a subject for another study<sup>114</sup>.

#### ABSTRACT

The purpose of this study is to investigate Duns Scotus’s uses of the technique of *positio impossibilis* for elaborating his concept of *ens infinitum* or ontological infinitas in *Quodlibet* q. 5. Although there are several studies about Scotus’s account of infinite being as a proper concept of God and of *infinitum* as the fundamental intrinsic mode of being, there is no targeted investigation of his conscious application of that logical tool for the purpose of concept formation. In order to acquire a complex concept like “infinite being”, Scotus is concerned first of all with the compatibility of its contents and with conceivability. He starts the technique exploring mathematical potential infinity according to Aristotle and arrives *secundum imaginationem* at ontological actual infinity through the analysis of the notions of “whole” (*totum*) and “perfection” (*perfectum*).

<sup>114</sup> I am very grateful to Prof. Gérard Sondag and to Prof. Simo Knuuttila for their comments on earlier drafts of this essay. I thank also Prof. Martin Tracey for correcting the English version of it.